

Evolution and Game Theory

① Influence of GT on bio animal behavior  
 Strategies  $\leftrightarrow$  genes  
 payoffs  $\leftrightarrow$  genetic fitness

« good strategies "grow"  
 but the strategies are not chosen »  
 hard-wired

② Influence from bio  $\rightarrow$  social sciences  
 « firms with rules of thumb decisions,  
 and markets selecting/surviving the fittest »

Simplified Model

- within species competition
- symmetric 2-player games
- large pop, random matching - avg payoffs
- relatively successful strategies grow

no gene redistribution  
 asexual reproduction

	C	D
C coop	2, 2	0, 3
D defect	3, 0	1, 1

1- $\epsilon$   $\epsilon$  (for C being majority)  
 $\epsilon$  1- $\epsilon$  (for D being majority)

e.g. lions on a hunt  
 ants defending a nest

Is cooperation Evolutionarily Stable?

C vs  $[(1-\epsilon)C + \epsilon D] \rightarrow (1-\epsilon)[2] + \epsilon[0] = 2(1-\epsilon)$

D vs  $[(1-\epsilon)C + \epsilon D] \rightarrow (1-\epsilon)[3] + \epsilon[1] = 3(1-\epsilon) + \epsilon$

so conclude C is not ES (evolutionarily stable)

Open Yale courses

Is D ES?

D vs  $[(1-\epsilon)D + \epsilon C] \rightarrow (1-\epsilon)[1] + \epsilon[3] = (1-\epsilon) + 3\epsilon$

C vs  $[(1-\epsilon)D + \epsilon D] \rightarrow (1-\epsilon)[0] + \epsilon[2] = 2\epsilon$

D is ES (any mutation from D gets wiped out)

Lesson ① Nature can suck

« sexual reproduction can change this »

② If a strategy is strictly dominated then it is not ES.

« the strictly dominant strategy will be a successful mutation »

	a	b	c
a	2, 2		
b			1, 1
c		0, 1	

Is c ES? - No

c vs  $[(1-\epsilon)c + \epsilon b] \rightarrow (1-\epsilon)[0] + \epsilon[1] = \epsilon$

b vs  $[(1-\epsilon)c + \epsilon b] \rightarrow (1-\epsilon)[1] + \epsilon[0] = 1 - \epsilon$

« b will grow from small proportion ( $\epsilon$ ) to  $\frac{1}{2}$  »  
 b/c  $\epsilon$  small

• Note: b, the invader, is itself not ES  
 « but it still avoids dying out »

Is c a NE?

No, because b is a profitable deviation

Lesson If s is not Nash,  
 (s,s) is not NE,  
 Then S is not ES.



If s is ES  $\Rightarrow$  (s,s) is NE

	a	b
a	1, 1	0, 0
b	0, 0	0, 0
	ε	1-ε

NE = (a,a), (b,b)

Is b ES?

$$b \rightsquigarrow [0] = 0$$

$$a \rightsquigarrow (1-\epsilon)[0] + \epsilon[1] = \epsilon$$

<< so b, b was Nash, but was not ES >>

<< reason is because b is a weak Nash >>

If (s,s) is a strict NE,

then s is ES

Fix a  $\hat{s}$  and suppose  $(\hat{s}, \hat{s})$  is NE  
 ↳ ie  $u(\hat{s}, \hat{s}) \geq u(\hat{s}, s')$  for all  $s'$

Two cases

(a)  $u(\hat{s}, \hat{s}) > u(\hat{s}, s')$  for all  $s'$

the mutant dies out because she meets  $\hat{s}$  oft.

(b)  $u(\hat{s}, \hat{s}) = u(\hat{s}, \hat{s})$  but

$u(\hat{s}, s') > u(\hat{s}, \hat{s})$

the mutant does "okay" against  $\hat{s}$  (the masses)  
 but badly against  $s'$  (itself)

ie:

<< (a) The mutant does poorly against the masses >>

>> (b) The mutant does equally against the masses  
 but gets clobbered against itself

1) Formal Definition (BIO - Maynard Smith 1972)

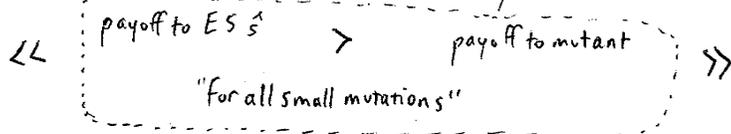
BIO

In a symmetric, 2 player game,  
 the pure strategy  $\hat{s}$  is ES (in pure strategies) if

there exists an  $\bar{\epsilon} > 0$

$$(1-\epsilon)[u(\hat{s}, \hat{s})] + \epsilon[u(\hat{s}, s')] > (1-\epsilon)u(\hat{s}, \hat{s}) + \epsilon u(s', s')$$

for all possible deviations  $s'$   
 and for all mutation sizes  $\epsilon < \bar{\epsilon}$



2) In a symmetric, 2 player game,

ECON

A strategy  $\hat{s}$  is ES (in pure strategies) if

(a)  $(\hat{s}, \hat{s})$  is a (symmetric) NE  
 ie  $u(\hat{s}, \hat{s}) \geq u(\hat{s}, s')$  for all  $s'$

AND

(b) if  $u(\hat{s}, \hat{s}) = u(\hat{s}, s')$   
 then  $u(\hat{s}, s') > u(s', s')$  ] "it better beat up on the mutant"  
 "you're better against the mutant than it is against itself"