

# Problem Set 1

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Four Questions due September 19, 2007.

**1. Strictly and Weakly Dominated Strategies?** What is the definition of a strictly dominated strategy? What is the definition of a weakly dominated strategy? Give an example of a two-player game matrix where one player has three strategies, one of which is strictly dominated; and the other player has three strategies, one of which is weakly (but not strictly) dominated. Indicate the dominated strategies.

**2. Iterative Deletion of (weakly) Dominated Strategies** Consider the following two-player game

		2		
		<i>l</i>	<i>c</i>	<i>r</i>
1	<i>T</i>	1, 1	0, 1	3, 1
	<i>M</i>	1, 0	2, 2	1, 3
	<i>D</i>	1, 3	3, 1	2, 2

(a) Are there any strictly dominated strategies? Are there any weakly dominated strategies? If so, explain what dominates what and how.

(b) After deleting any strictly or weakly dominated strategies, are there any strictly or weakly dominated strategies in the ‘reduced’ game? If so, explain what dominates what and how. What is left?

(c) Go back to your argument for deleting in the first ‘round’ and recall what dominated what and how. Compare this with what was deleted in the ‘second’ round. Comment on how this might make you a bit cautious when iteratively deleting weakly dominated strategies?

**3. Hotelling’s Location Game.** Recall the voting game we discussed in class. There are two candidates, each of whom chooses a position from the set  $S_i := \{1, 2, \dots, 10\}$ . The voters are equally distributed across these ten positions. Voters vote for the candidate whose position is closest to theirs. If the two candidates are equidistant from a given position, the voters at that position split their votes equally. The aim of the candidates is to maximize their percentage of the total vote. Thus, for example,

$u_1(8, 8) = 50$  and  $u_1(7, 8) = 70$ . [**Hint**: in answering this question, you do **not** need to write out the full payoff matrices!]

(a) In class, we showed that strategy 2 strictly dominates strategy 1. In fact, other strategies strictly dominate strategy 1. Find all the strategies that strictly dominate strategy 1. Explain your answer. [**Hint**: try some guesses and see if they work.]

(b) Suppose now that there are three candidates. Thus, for example,  $u_1(8, 8, 8) = 33.\dot{3}$  and  $u_1(7, 9, 9) = 73.\dot{3}$ . Is strategy 1 dominated, strictly or weakly, by strategy 2. How about by strategy 3? Explain. Suppose we delete strategies 1 and 10. That is, we rule out the possibility of any candidate choosing either 1 or 10, although there are still voters at those positions. Is strategy 2 dominated, strictly or weakly, by any other (pure) strategy  $s_i$  in the reduced game? Explain.

4. “**Strength can be weakness**”. A three-person committee has to choose a winner for a national art prize. After some debate, there are three candidates still under consideration: a woman who draws antelope in urban settings, a man who makes rectangular lead boxes, and a woman who sculpts charcoal. Lets call these candidates  $a$ ,  $b$  and  $c$ ; and call the committee members 1,2 and 3. The preferences of the committee members are as follows: member 1 prefers  $a$  to  $b$  and  $b$  to  $c$ ; member 2 prefers  $c$  to  $a$  and  $a$  to  $b$ ; and member 3 prefers  $b$  to  $c$  and  $c$  to  $a$ . The rules of the competition say that, if they disagree, they should vote (secret ballot, one member one vote) and that, if and only if the vote is tied, the winner will be the candidate for whom member 1 voted. Thus, it might seem that member 1 has an advantage.

(a) Consider this voting game. Each voter has three strategies:  $a$ ,  $b$ , or  $c$ . For each voter, which strategies are weakly or strictly dominated? [**Hints**. Be especially careful in the case of voter 1. To answer this question, you do **not** need to know the exact payoffs: any payoffs will do provided that they are consistent with the preference orders given above. To answer this question, you do **not** have to write out matrices].

(b) Now consider the reduced game in which all weakly and strictly dominated strategies have been removed. For each voter, which strategies are now weakly or strictly dominated? What is the predicted outcome of the vote? Compare this outcome to voter 1’s preferences and comment.