Answers to Practice Problems on Asymmetric Information

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Dec 2007

(1) Entry Deterrence (Harbaugh). [Consider a two-period game. In the first period, an incumbent monopolist sets the price for its product. In the second period a potential entrant might enter the market or might stay out. The incumbent is either low cost or high cost. The entrant does not know which: the incumbent could be low or high cost with equal probability. In the first period, if the incumbent chooses to have a high price, it earns 200 if it is low cost or 100 if it is high cost. If it chooses a low price, it earns 150 if it is low cost or 0 if it is high cost. While it hurts profits in the first period, setting a low price might discourage entry and boost profits in the second period. In the second period, if there is no entry, the incumbent will choose the high price and get 200 or 100 depending on its type. If there is entry, however, competition will force the incumbent to get only 50 if it is the low-cost type and only 10 if it is the high-cost type. The entrant must make an unrecoverable investment if it decides to enter the market. Its profits are -25 if the incumbent is low cost and 50 if the incumbent is high cost. The incumbent first decides the first-period price, then the entrant makes her decision.

(a) Consider a putative *separating* equilibrium in which the incumbent would charge the low price in the first period if it is the low-cost type and would charge the high price if it were the high-cost type. Show that this is in fact an equilibrium. In particular, what does the potential entrant believe about the incumbent when seeing each price, and does it enter; and could either type of incumbent do better by deviating in the first period.

Answer: *To be consistent with the equilibrium actions, the potential entrant must believe that the incumbent is low cost if she sees a low price in the first period and believe that the incumbent is high cost if she sees a high price. In this case, she should enter if and only if she sees a high first-period price. Given this, if the low-cost incumbent sticks to the equilibrium, she makes 150 in the first period and 200 in the second period. If she deviates to a high first-period price, she gets 200 in the first period but (since she would be identified as high cost) only 50 in the second. Clearly, she won't deviate. If the high-cost incumbent sticks to the equilibrium, she gets 100 in the first period and 10 in the second period. If she deviated to the low price in the first period, she would get 0 the first period and (since she would be identified as low cost) 100 in the second. Clearly, she won't deviate either. So, this is an equilibrium.*

(b) Consider a perverse putative *separating* equilibrium in which the incumbent would charge the high price in the first period if it is the low-cost type and would charge the low price if it were the **high** cost type. Is this an equilibrium? Explain.

Answer: *To be consistent with the perverse putative equilibrium actions, the potential entrant*

must believe that the incumbent is high cost if she sees a low price in the first period and believe that the incumbent is low cost if she sees a high price. In this case, she should enter if and only if she sees a low first-period price. Given this, if the high-cost incumbent sticks to the equilibrium, she gets 0 in the first period and 10 in the second period. If she deviated to the high price in the first period, she would get 100 the first period and (since she would be identified as low cost) 100 in the second. Clearly, she will deviate.

(2) In the News. Hillary is considering running for president. A private opinion poll has told her whether she would have a *high* or *low* chance of winning the general election assuming she is her party's candidate. She has no way to communicate the results of this poll verifiably to other potential candidates: polling numbers are too easy to fabricate. After observing the poll, however, Hillary can choose whether or not to write another book with her husband. Regardless of Hillary's chance of winning, writing the book would cost Hillary 1 util. Think of this as loss of self-esteem.

Al gets to decide whether or not to enter the primary after observing whether or not Hillary writes the book. Hillary would prefer Al to stay out of the primary race. Al staying out is worth 4 utils to Hillary if Hillary would have a *high* chance of winning the general election; and Al staying out is worth 2 utils to Hillary if Hillary would have a *low* chance of winning the general election. If Al runs, Hillary gains 0 utils regardless of his chances.

Al only wants to run if Hillary's chances in the general election are bad. Al gets 2 utils from running if Hillary would have a *low* chance of winning the general election; and Al gets -1 utils from running if Hillary would have a *high* chance of winning the general election. If Al stays out then his payoff is 0. Initially, before seeing whether or not Hillary writes a book, Al thinks Hillary is equally likely to have a *low* or *high* chance of winning the general election.

The following table may be helpful.

	high-chance Hillary	low-chance Hillary
Hillary's book cost	1	1
Hillary's gain if Al stays out	4	2
Hillary's gain if Al runs	0	0
Al's payoff if he runs	-1	2
Al's payoff if he stays out	0	0
Al's initial belief	$\frac{1}{2}$	$\frac{1}{2}$

(a) Is there a separating pure-strategy equilibrium in which Hillary makes a different decision whether or not to write the book depending on whether he has a *high* or *low* chance of winning the general election. If such equilibria exist, describe one and show it is an equilibrium. If no such equilibrium exists, explain why not.

Answer: *Notice that Hillary would like Al not to run, and that Al would not like to run if Hillary has a high chance of winning. The question is asking whether writing the book is sufficient a signal to separate whether Hillary has high or low chances of winning. It is easy to see that it is not an equilibrium for Hillary only to write the book if he has a low chance of winning. So consider the supposed equilibrium in which Hillary only writes if he has a high chance of winning. In this case, if Hillary has a low chance of winning, then he does not write the book, he*

is recognized as having a low chance of winning and Al enters, leading to a payoff for Hillary of 0. If 'low-chance Hillary' were to write the book then Al would think Hillary had a high chance of winning and stay out. In this case, low-chance Hillary would get a payoff of $2 - 1$. *Thus low-chance Hillary will deviate.*

Now suppose that Hillary can still write the book or do nothing, but he also has a third choice. If he does not write the book, he can go on Saturday Night Live. This is likely to be a humiliating experience: its cost to Hillary (regardless of his chance of winning the election) is 3 utils.

(b) Is there a separating pure-strategy equilibrium in which Hillary makes a different decision whether to go on Saturday Night Live, write a book or do nothing depending on whether he has a *high* or *low* chance of winning the election. If such equilibria exist, describe them and show they are equilibria. If no such equilibrium exists, explain why not.

Answer: *Consider a possible equilibrium in which Hillary goes on Saturday Night Live if and only if he has a high chance of winning, Hillary never writes the book regardless of his chances, and Al stays out if and only if Al goes on Saturday Night Live. To be consistent with Hillary's equilibrium choices, Al's beliefs must identify Hillary as having a high chance of winning if and only if he goes on the show, so Al's decision makes sense given his belief. High-chance Hillary could deviate and not go on the show but this would lead to Al's running. Such a deviation would save* 3 *but it would cost him* 4. *Meanwhile low chance Hillary could deviate and go on the show and hence induce Al not to run. But such a deviation would gain him* 2 *at a cost of* 4. *Neither type of Hillary benefit by writing the book since it does not deter Al's entering the race. Thus this is an equilibrium.*

(3) (*Hard*) **Nobility Doesn't Advertise**. Imagine a country where each person is either upper class, middle class or lower class, and there is an equal proportion of each type. Regardless of one's true class, being thought to be upper class is worth 120; being thought to be middle class is worth 90; and being thought to be lower class is worth 0. One way in which a person can try to indicate their class to others is by choosing 'to signal'. This signal is a crude 'yes-or-no' choice; that is, each person can only choose whether to signal or not. The cost of the signal is 200 for lower-class types; 10 for middle-class types; and 0 for upper-class types. Whether or not a person chooses to signal, after he or she has made that decision, he or she must take a compulsory 'high-society' test. The outcome of the test is completely independent of whether or not a person has signaled. Upper-class types always pass the test; lower-class types never pass; and middle-class types pass with probability one half.

(a) Suppose there were an equilibrium in which only the middle class signaled. In such an equilibrium, what must society believe when it sees a person signaling and passing the test; signaling and failing; not signaling and passing; or not signaling and failing?

Answer: *Beliefs must be consistent with the equilibrium. So, in equilibrium, (signal, pass) and (signal, fail) means the person is middle class: since only middle's signal. Similarly, (no Signal, pass) means the person is upper class; and (no signal, fail) means the person is lower class.*

(b) Show carefully that there is indeed an equilibrium in which only middle-class types signal.

Answer: *We need to check the incentive conditions: that is, each type does better doing what it is doing than by deviating. In this equilibrium, the upper class get* 120 *if they do not signal, which is greater than 90 (since they are mistaken for middle class) if they signal. The lower class get* 0 *if they do not signal which is better than* $90 - 200 = -110$ *if they signal (and are mistaken for middle class). By signaling, the middle class get* $90 - 10 = 80$. *If they do not signal then, with probability* 0.5 *they pass (and so get mistaken for upper class) and get* 120*; and with probability* 0.5 *they fail (and so get mistaken for lower class) and get* 0. *Thus, the expectation from not signaling if you are middle class is* 60. *So the middle class are happy to signal.*

(c) [hard] Show that there is also an equilibrium in which both middle- and upper-class types signal. To do this, you may find it helpful to assume society believes that anyone who does not signal and who passes the test is middle class.

Answer: *In equilibrium, the probability that a person is (upper class, signals and passes) is* 1/3 *since all upper class people signal and pass. The probability that a person is (middle class, signals and passes) is* 1/6 *since all middle class people signal but only half of them pass. So (by Bayes rule), the probability that a person who signals and passes is upper class is* $(1/3)/(1/3 + 1/6) = 2/3$*; and the probability that they are middle class is* $(1/6)/(1/3+1/6) = 1/3$. *That is, if society sees someone signal and pass then society thinks that person is upper class with probability* 2/3 *and middle class with probability* 1/3. *Similarly, in this putative equilibrium, If society sees someone signal and fail, then they must be middle class; and non-signalers must be lower class.*

Now lets check deviations. In equilibrium, the upper class get $2/3(120) + 1/3(90) = 110$. *If an upper class person deviates and does not signal, he still passes and so gets* 90 *(by our assumption). So the upper class are happy to signal. In equilibrium, a middle class person gets* $0.5[2/3(120) + 1/3(90)] + 0.5[90] - 10$ *where the first bracket is if he passes and the second is if he fails. If he deviates and does not signal then he gets* $0.5[90] + 0.5[0]$ *where the first bracket is if he passes (and so is thought middle class by our assumption) and the second is if he fails (and so is thought lower class). The former total* 90 *whereas the latter is* 45*, so the middle class is happy to signal. Finally, since the cost of signaling is* 200 *if you are lower class, nothing could ever induce you to signal.*

(4) An Auction. Alice and Bob would both like to own the same hand-written and signed manuscript of a Frost poem. The manuscript is worth \$5 million to Alice and worth \$3 million to Bob. The present owner of the manuscript proposes the following method of sale, known as a 'second-price auction'. Alice and Bob will each simultaneously write down a 'bid' for the manuscript. Let b_A be Alice's bid, and let b_B be Bob's bid. The manuscript will go to the person whose bid is highest, and that person will have to pay **the other person**'s bid. If the bids are tied, then a fair coin will be tossed to decide who gets the manuscript, and that person will have to pay the tied bid. In either case, the person who does not get the manuscript pays nothing.

Here then is Alice's payoff function in this game (in \$ millions):

$$u_A(b_A, b_B) = \begin{cases} 5 - b_B & \text{if } b_A > b_B \\ 0 & \text{if } b_B > b_A \\ \frac{1}{2}(5 - b_B) & \text{if } b_A = b_B \end{cases}$$

Everything above is common knowledge to the players (in particular, they know each other's

payoffs). There are no other bidders. Negative bids are not allowed. [*Notice that you do not need to answer parts (c) and (d) in order to answer part (e).*]

(a) Write down Bob's payoff function $u_B(b_A, b_B)$.

Answer: *Bob's payoff function is*

$$u_B(b_A, b_B) = \begin{cases} 3 - b_A & \text{if } b_B > b_A \\ 0 & \text{if } b_A > b_B \\ \frac{1}{2}(3 - b_B) & \text{if } b_A = b_B \end{cases}$$

(b) What are Alice's best responses to $b_B = 2$? What are Alice's best responses to $b_B = 10$?.

Answer: *In the first case, Alice will pay* 2 *if she wins the auction. Since the good is worth* 5 *to her, she wants to win. Thus any bid greater than* 2 *is a best response. In the latter case, she would pay* 10 *she wins. Thus any bid less than 10 (to ensure she loses) is a best response.*

(c) Are the following strategy profiles equilibria of the game: $(5, 3)$; $(4, 7)$; $(2, 3)$; $(10, 0)$ where the first number is b_A and the second is b_B. Explain your answer.

Answer: *The profile* $(5, 3)$ *is an equilibrium: Alice's payoff is* $5 - 3 = 2$*, and Bob's is* 0*. If Alice deviates by some* $b'_A > 3$*, it make no difference to her payoff. If she deviates by some* $b'_A < 3$*, she gets* 0*. If she deviated by bidding* 3 *she would get a payoff of* 1*. If Bob deviates by bidding equal or more than* 5*, then he gets a negative payoff. If he deviates to a bid less than* 5*, his payoff is still* 0*.*

The profile $(4, 7)$ *is not an equilibrium. For example, Bob's payoff here is* $3 - 4 = -1$*, he wins the good but pays 'too much' for it. He can deviate to (say)* 0 *and get a payoff of* 0*.*

The profile $(2, 3)$ *is not an equilibrium. For example, Alice's payoff here is* 0*. She can deviate to (say)* 9 *and get a payoff of* $5 - 3 = 2$*.*

The profile $(10, 0)$ *is an equilibrium. Alice's payoff is* 5 *and Bob's is zero. No matter what she bids, provided it is greater than* 0 *she gets the same payoff. If she bids zero it makes her worse off. The only way for Bob to change his payoff is for him to bid more than* 10*. This would yield a negative payoff.*

(d) Are there any equilibria in which Bob wins the manuscript? Explain.

Answer: *The profile* $(0, 10)$ *is an equilibrium: Alice's payoff is* 0*, and Bob's is* 3*. The reason it is an equilibrium is the same as for* $(10, 0)$ *mutatis mutandis.*

(e) Which (if any) of Alice's and Bob's strategies are strictly dominated. Which (if any) are weakly dominated? Explain concisely but carefully.

Answer: *No strategy for either player is strictly dominated. For example, for any two bids* $b_A > b'_A$ *of Alice,* $u_A(b_A, b_B) = u_A(b'_A, b_B)$ *if* $b_B > b_A$*. But for each player, bidding her own value is weakly dominant. Let's just show this for Alice (the argument for Bob is similar). First consider a bid* $b'_A > 5$*.*

$u_A(b'_A, b_B)$ for $b'_A > 5$	$u_A(5, b_B)$	
$5 - b_B$	$5 - b_B$	if $b_B \leq 5$
negative	0	if $b'_A \geq b_B > 5$
0	0	if $5 > b'_A$

Next consider a bid $b''_A < 5$*.*

$u_A(b''_A, b_B)$ for $b''_A < 5$	$u_A(5, b_B)$	
0	0	if $b_B \geq 5$
0	$5 - b_B$	if $b'_A < b_B < 5$
$(5 - b_B)/2$	$5 - b_B$	if $b_B = b''_A$
$5 - b_B$	$5 - b_B$	if $b_B < b''_A$