The calculator problem and the evolutionary synthesis of arbitrary software

CREST Open Workshop on Genetic Programming for Software Engineering

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Tests

Software


STAPLES
SPL-110
12345678
%
OFF
ON/C
MRC
M-
M+
CE
7
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9
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Outline

- Arbitrary software
- Requirements and ways to meet them
- Tags, uniform variation, and lexicase selection
- The calculator problem
- Other problems and prospects

Arbitrary Software

- OS utilities
- Word processors
- Web browsers
- Accounting systems
- Image processing systems
- Everything

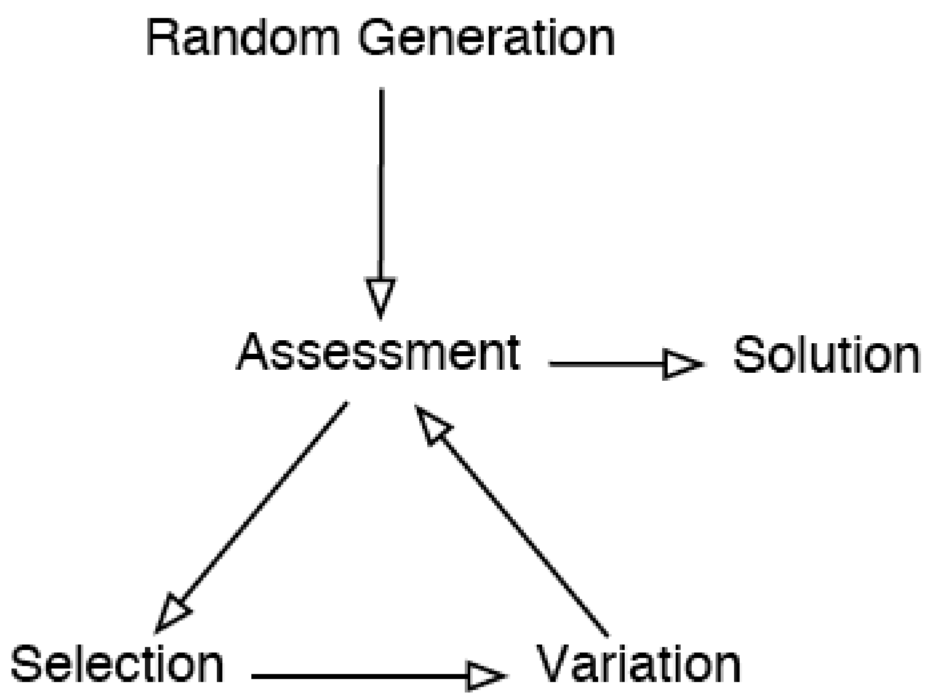
Arbitrary Software

- May be stateful, with multiple entry points
- May have a variety of interfaces involving a variety of types
- May require arbitrary Turing-computable functions
- Can be specified with behavioral tests

Requirements

- Represent and evolve arbitrary computable functions on arbitrary types (Push, uniform variation)
- Represent and evolve arbitrary computational architectures (modules; tags, tagged entry points)
- Drive evolution with performance tests (lexicase selection)

Evolutionary Computation



Genetic Programming

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software
- Potential (?): evolve software at all scales, including and surpassing the most ambitious and successful products of human software engineering

Program Representations

- Lisp-style symbolic expressions (Koza, ...).
- Purely functional/lambda expressions (Walsh, Yu, ...).
- Linear sequences of machine/byte code (Nordin et al., ...).
- Artificial assembly-like languages (Ray, Adami, ...).
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...).
- Graph-structured programs (Teller, Globus, ...).
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...)
- Fuzzy rule systems (Tunstel, Jamshidi, ...)
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...).
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, ...).

Evolvability

The fact that a computation *can* be expressed in a formalism does not imply that a correct program can be produced in that formalism by a human programmer or by an evolutionary process.

Data/Control Structure

- Data abstraction and organization

  Data types, variables, name spaces, data structures, ...

- Control abstraction and organization

  Conditionals, loops, modules, threads, ...

Structure via GP (1)

- Specialize GP techniques to directly support human programming language abstractions
- Strongly typed genetic programming
- Module acquisition/encapsulation systems
- Automatically defined functions
- Automatically defined macros
- Architecture altering operations

Structure via GP (2)

- Specialize GP techniques to indirectly support human programming language abstractions
- Constrain genetic change, or repair after genetic change, to satisfy abstraction syntax
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

Structure via GP (3)

- Forget about human programming abstractions (mostly)
- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- **Push**

Push

- A programming language developed specifically for evolutionary computation, as the language in which evolving programs are expressed
- Intended to maximize the evolvability of arbitrary computational processes

Push

- Stack-based postfix language with one stack per type
- Types include: integer, float, boolean, code, exec, vector, matrix, quantum gate, [add more as needed]
- Missing argument? NO-OP
- Minimal syntax:
  program → instruction | literal | (program*)

Why Push?

- Highly expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Evolvable
- Extensible
- Supports several forms of meta-evolution

Sample Push Instructions

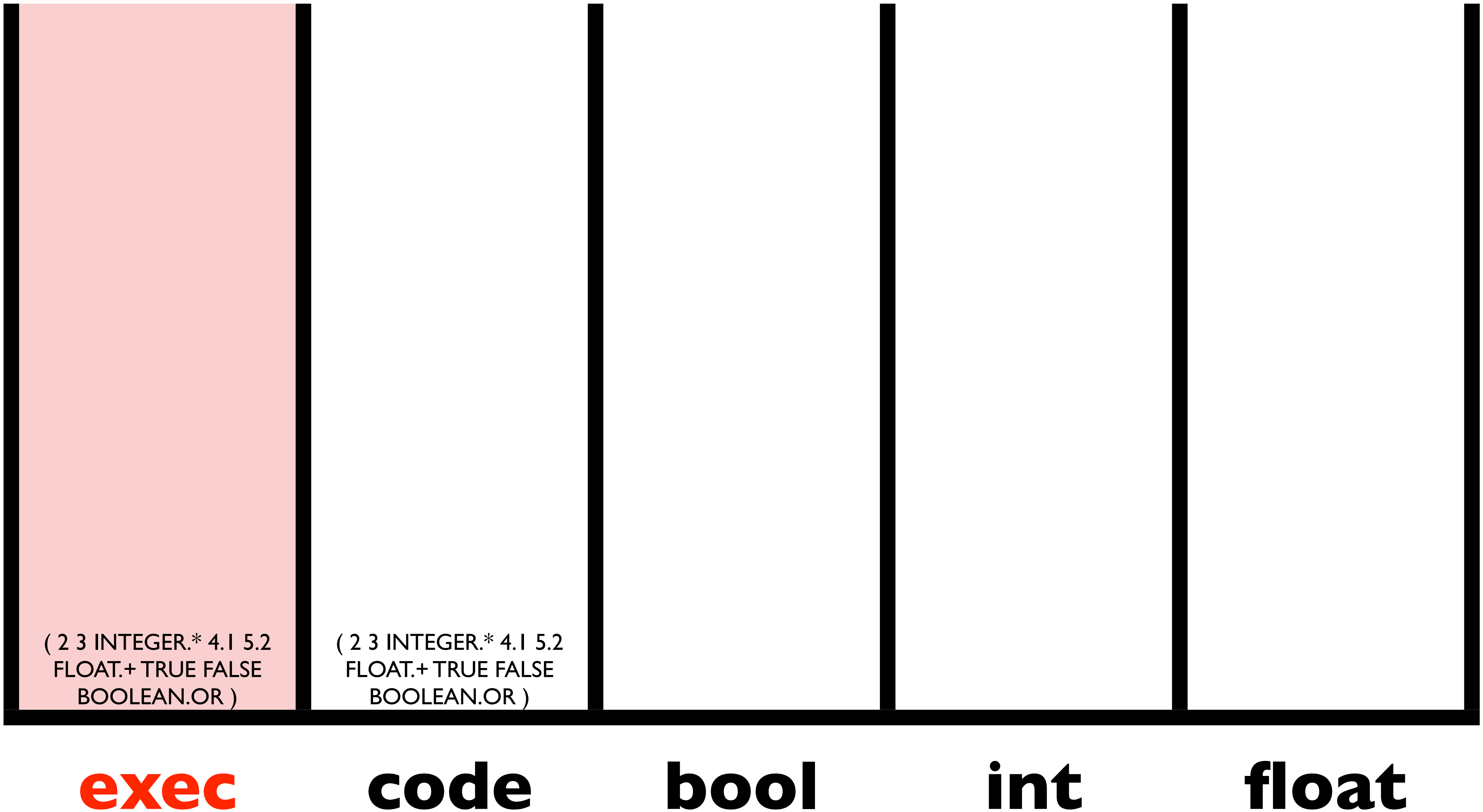
Stack manipulation instructions (all types)	POP, SWAP, YANK, DUP, STACKDEPTH, SHOVE, FLUSH, $=$
Math (INTEGER and FLOAT)	$+$, $-$, $/$, $*$, $>$, $<$, MIN, MAX
Logic (BOOLEAN)	AND, OR, NOT, FROMINTEGER
Code manipulation (CODE)	QUOTE, CAR, CDR, CONS, INSERT, LENGTH, LIST, MEMBER, NTH, EXTRACT
Control manipulation (CODE and EXEC)	DO*, DO*COUNT, DO*RANGE, DO*TIMES, IF

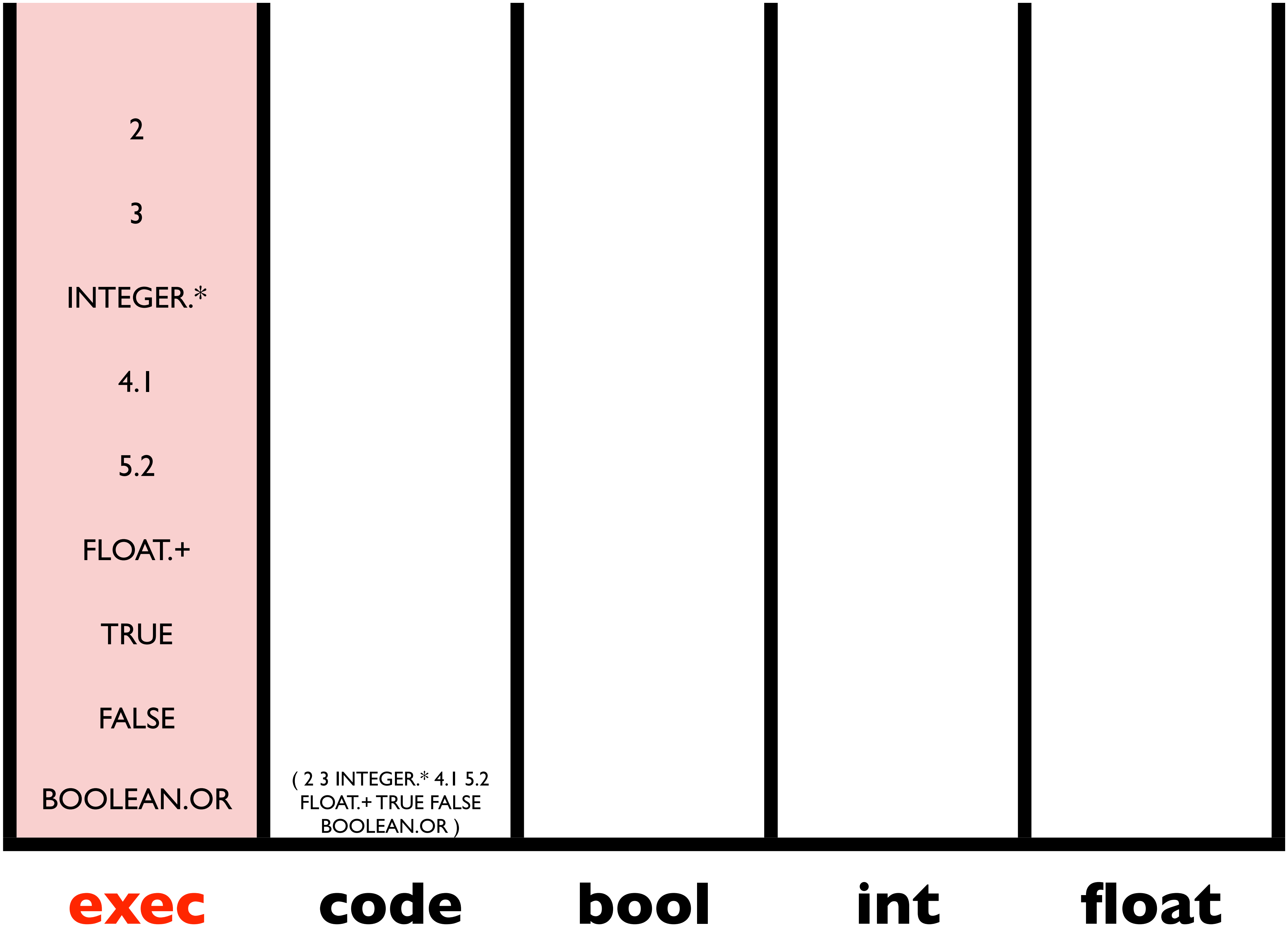
Push(3) Semantics

- To execute program P:

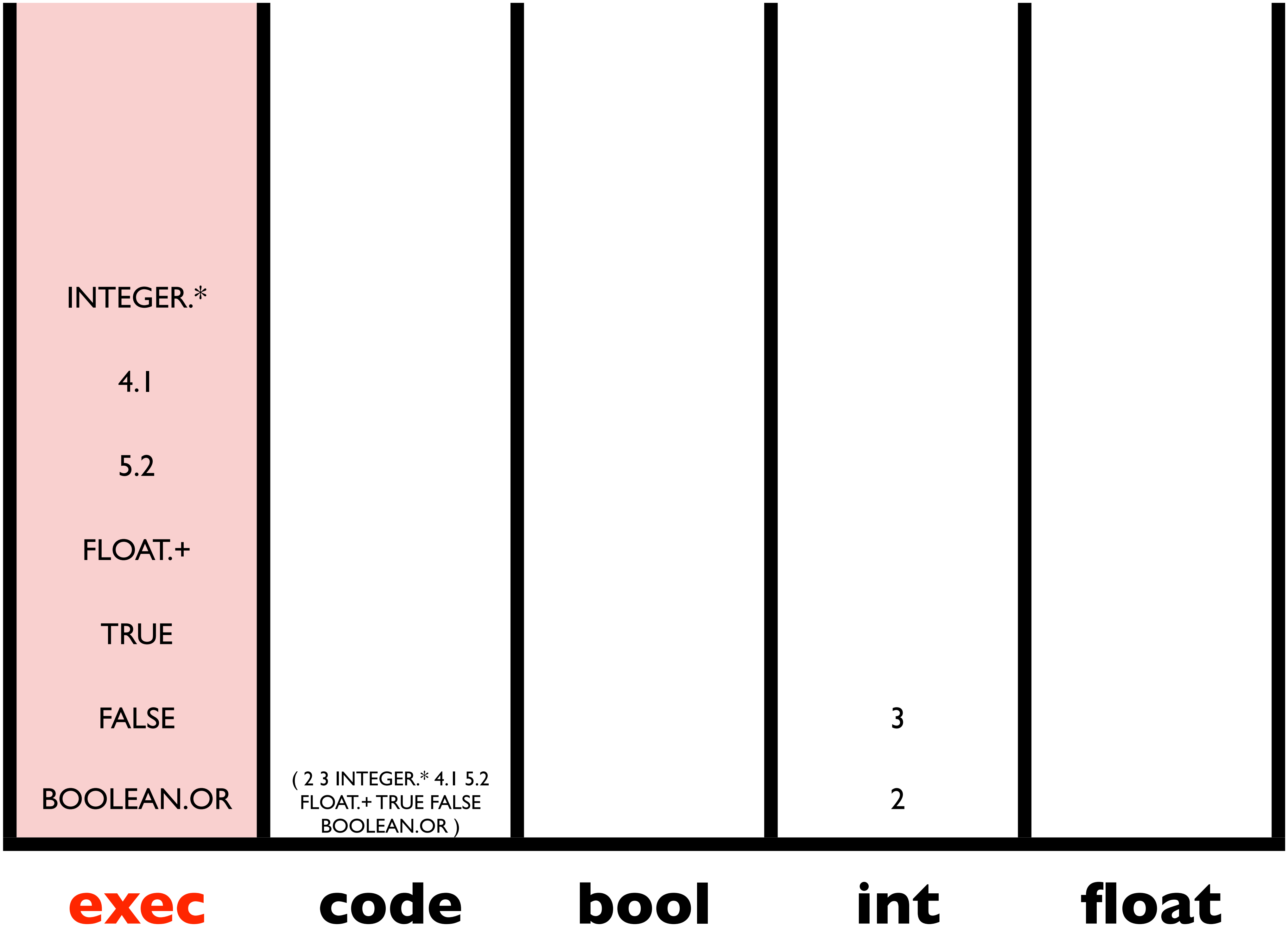
  1. Push P onto the **`EXEC`** stack.
  2. While the **`EXEC`** stack is not empty, pop and process the top element of the **`EXEC`** stack, E:
     (a) If E is an instruction: execute E (accessing whatever stacks are required).
     (b) If E is a literal: push E onto the appropriate stack.
     (c) If E is a list: push each element of E onto the **`EXEC`** stack, in reverse order.

```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+
TRUE FALSE BOOLEAN.OR )
```

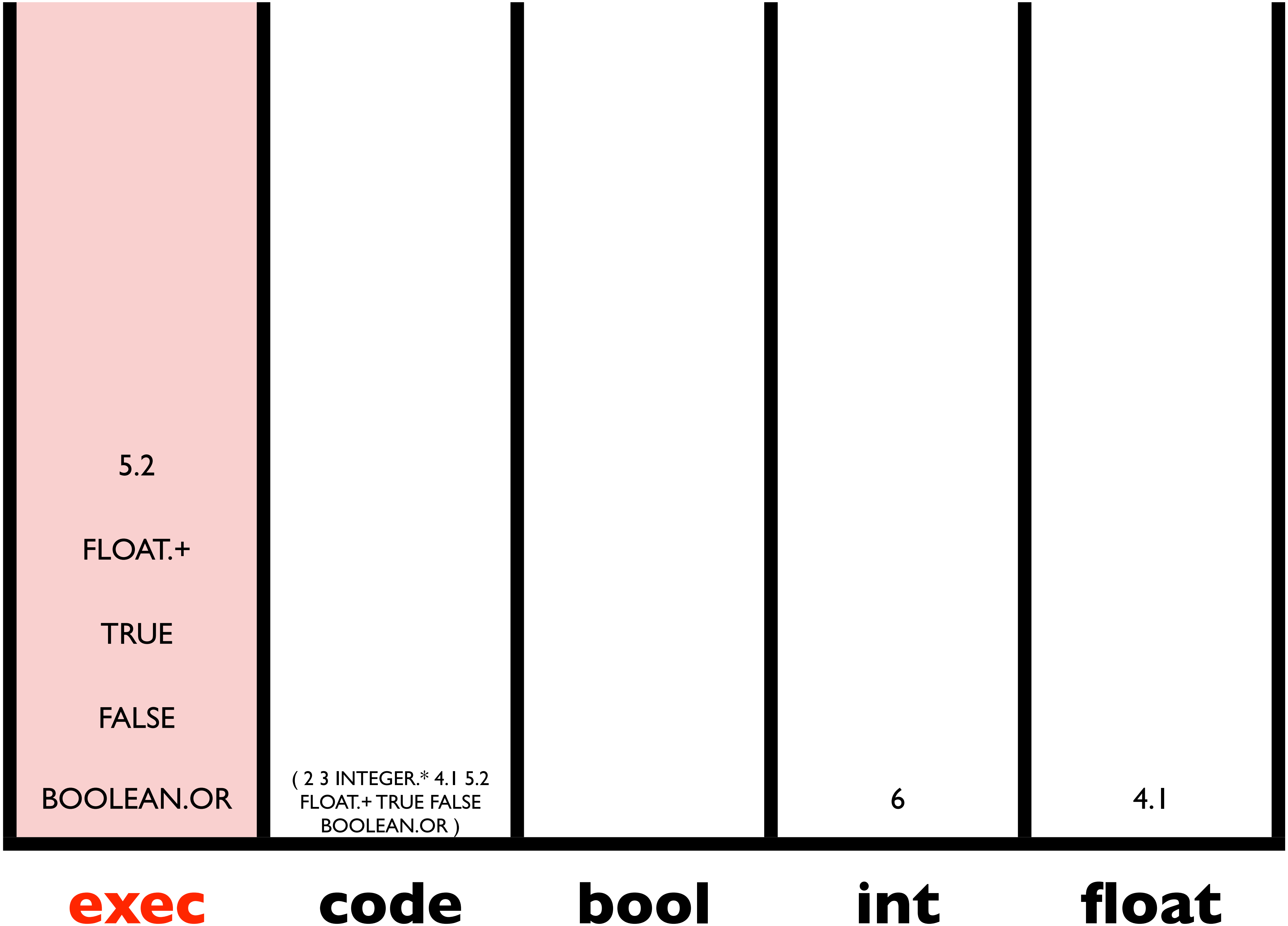



2
3
INTEGER.*
4.1
5.2
FLOAT.+
TRUE
FALSE
BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
exec
code
bool
int
float

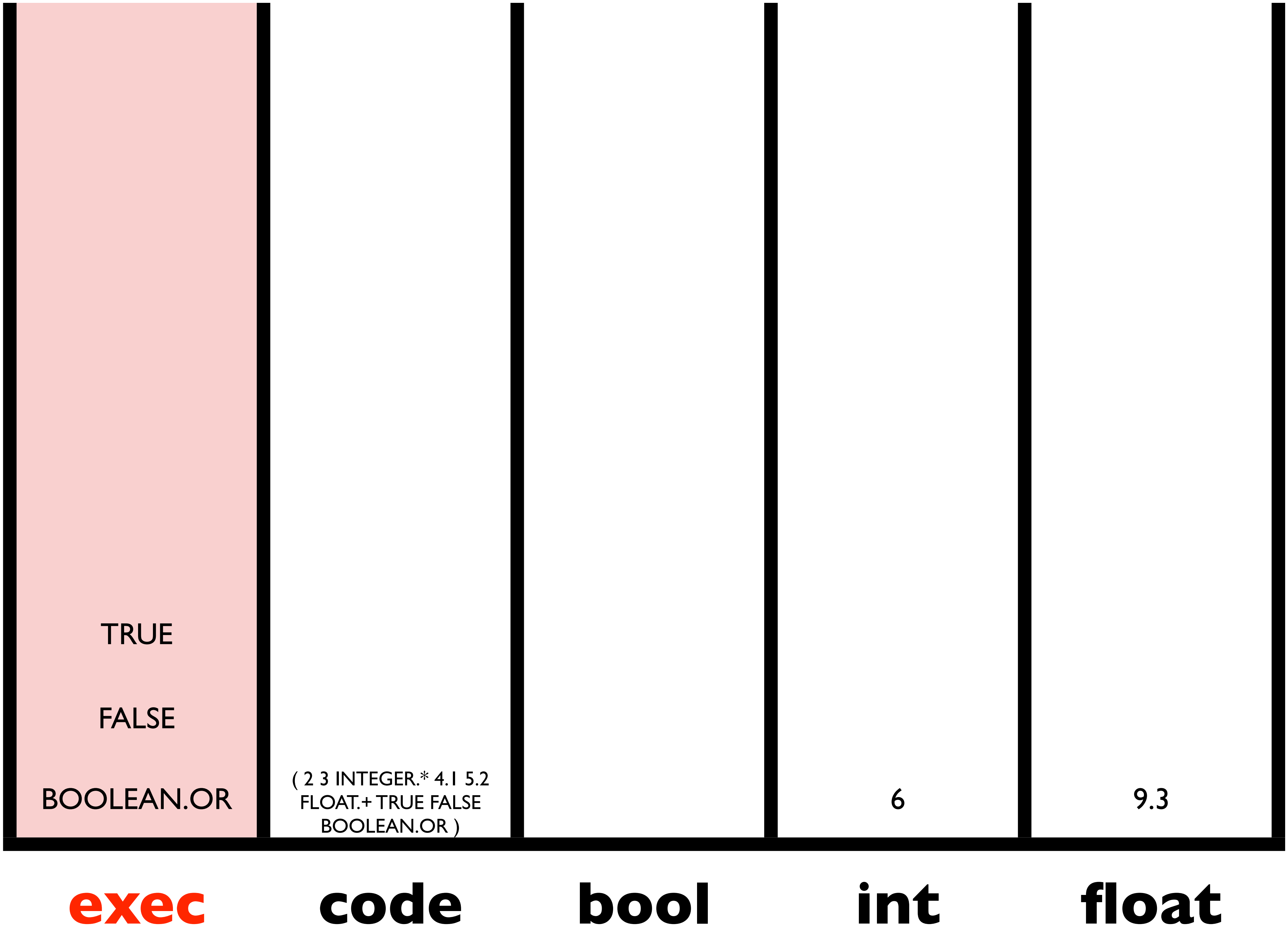
exec	code	bool	int	float
3				
INTEGER.*				
4.1				
5.2				
FLOAT.+				
TRUE				
FALSE				
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)		2	

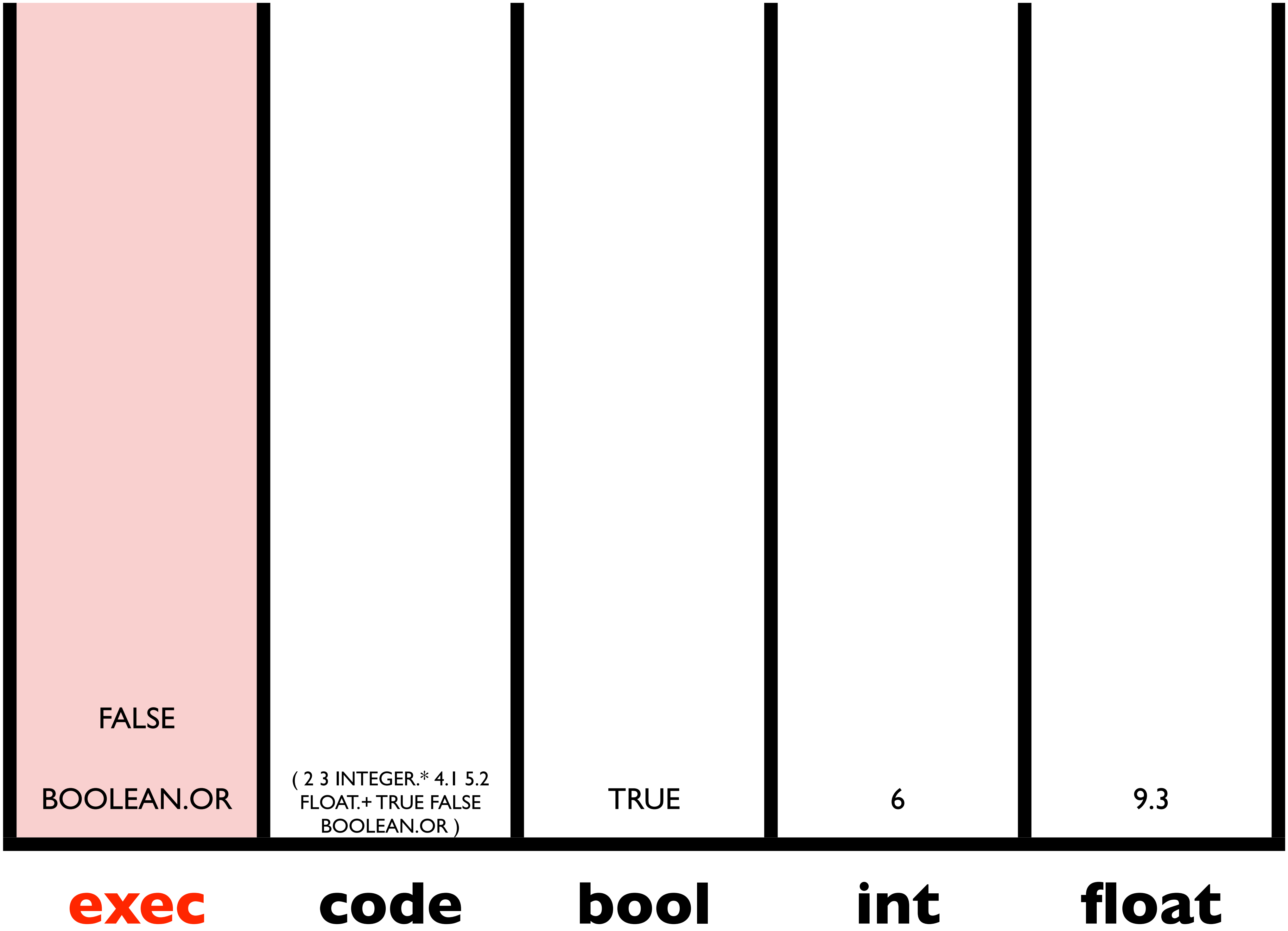

INTEGER.*
4.1
5.2
FLOAT.+
TRUE
FALSE
BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
3
2
exec
code
bool
int
float

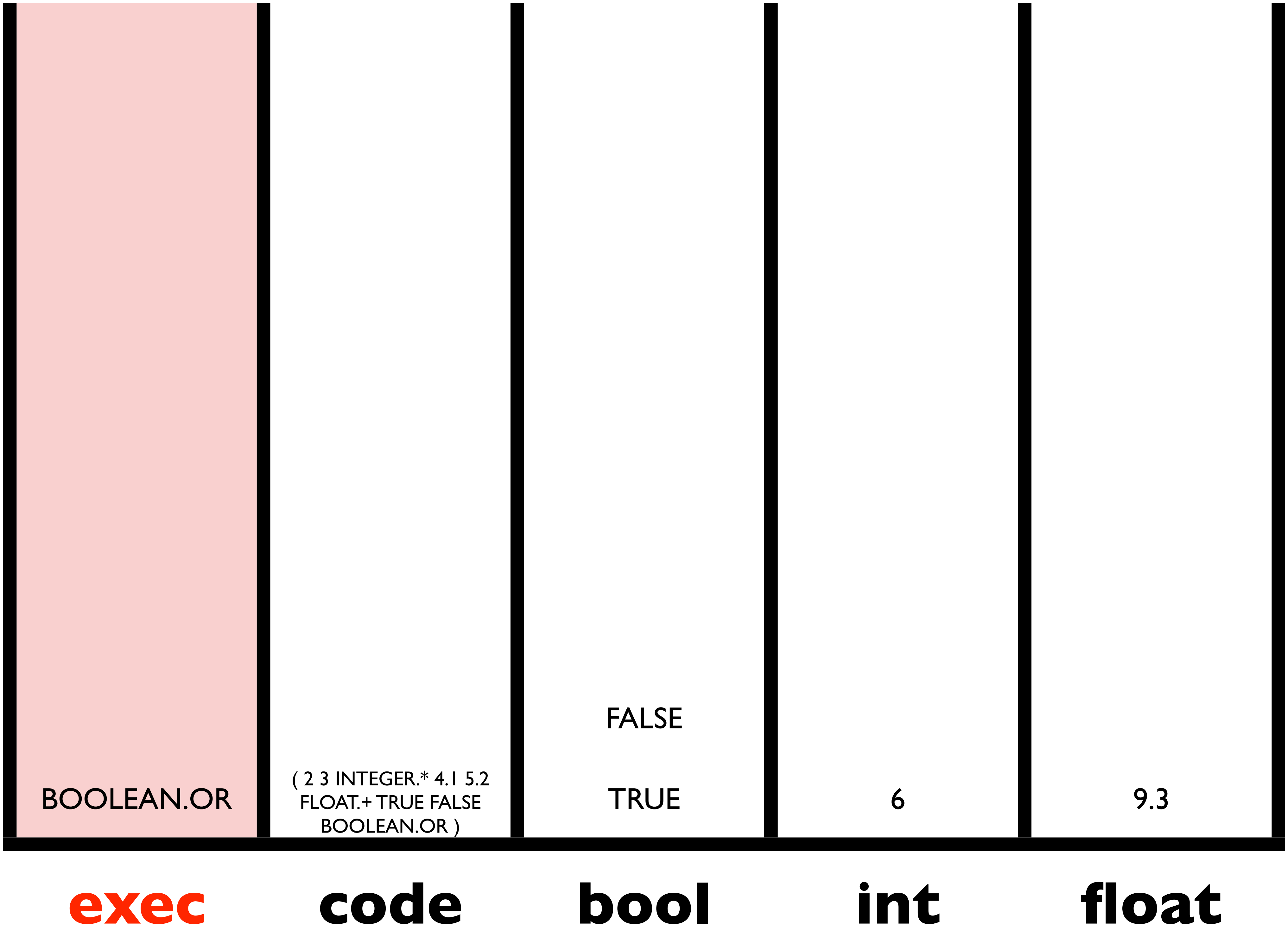
exec	code	bool	int	float
4.1				
5.2				
FLOAT.+				
TRUE				
FALSE				
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)		6	

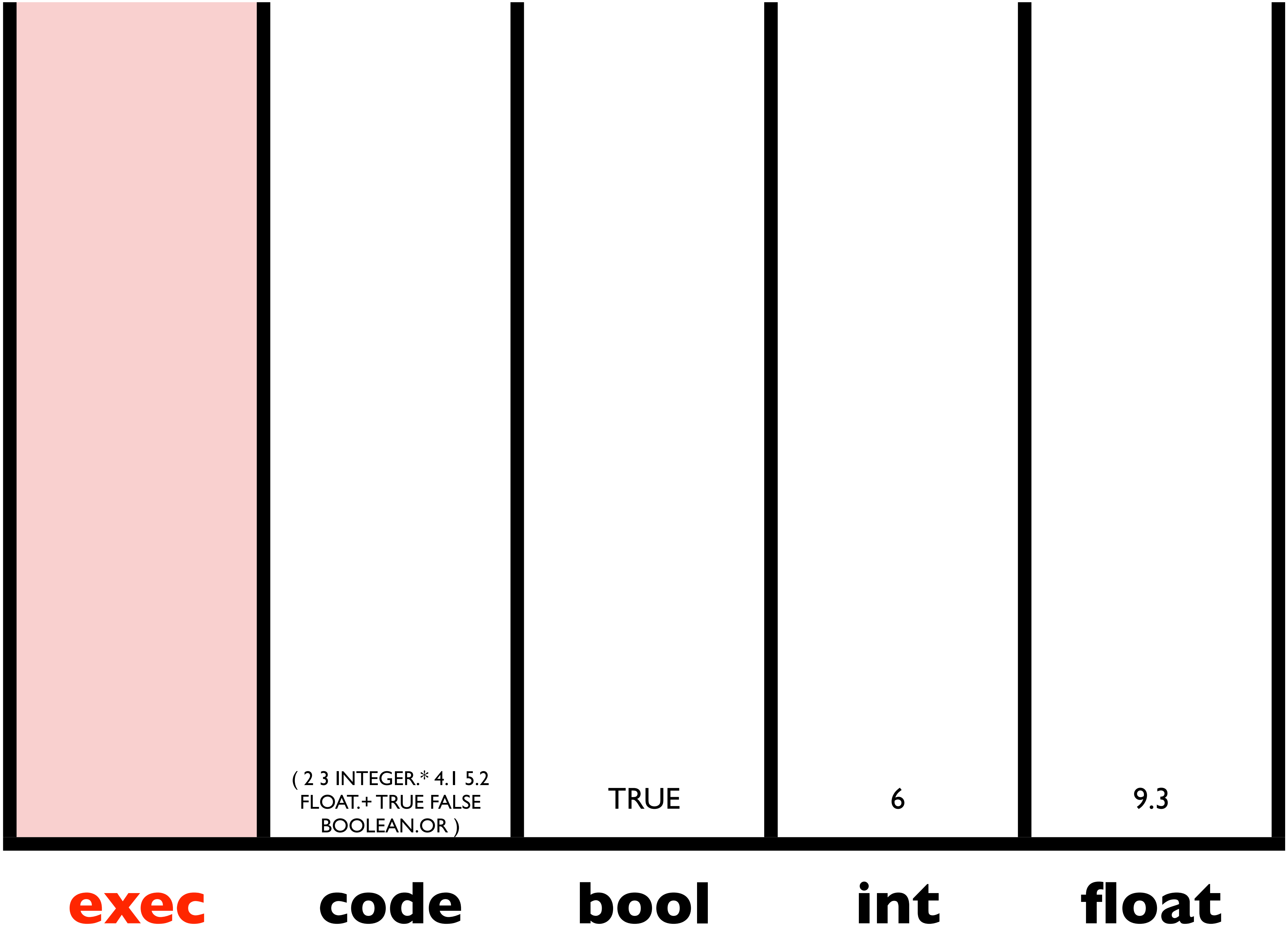

5.2
FLOAT.+
TRUE
FALSE
BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
6
4.1
exec
code
bool
int
float

exec	code	bool	int	float
FLOAT.+				
TRUE				
FALSE				5.2
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)		6	4.1


TRUE
FALSE
BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
6
9.3
exec
code
bool
int
float


FALSE
BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
TRUE
6
9.3
exec
code
bool
int
float


BOOLEAN.OR
(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
FALSE
TRUE
6
9.3
exec
code
bool
int
float


(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)
TRUE
6
9.3
exec
code
bool
int
float

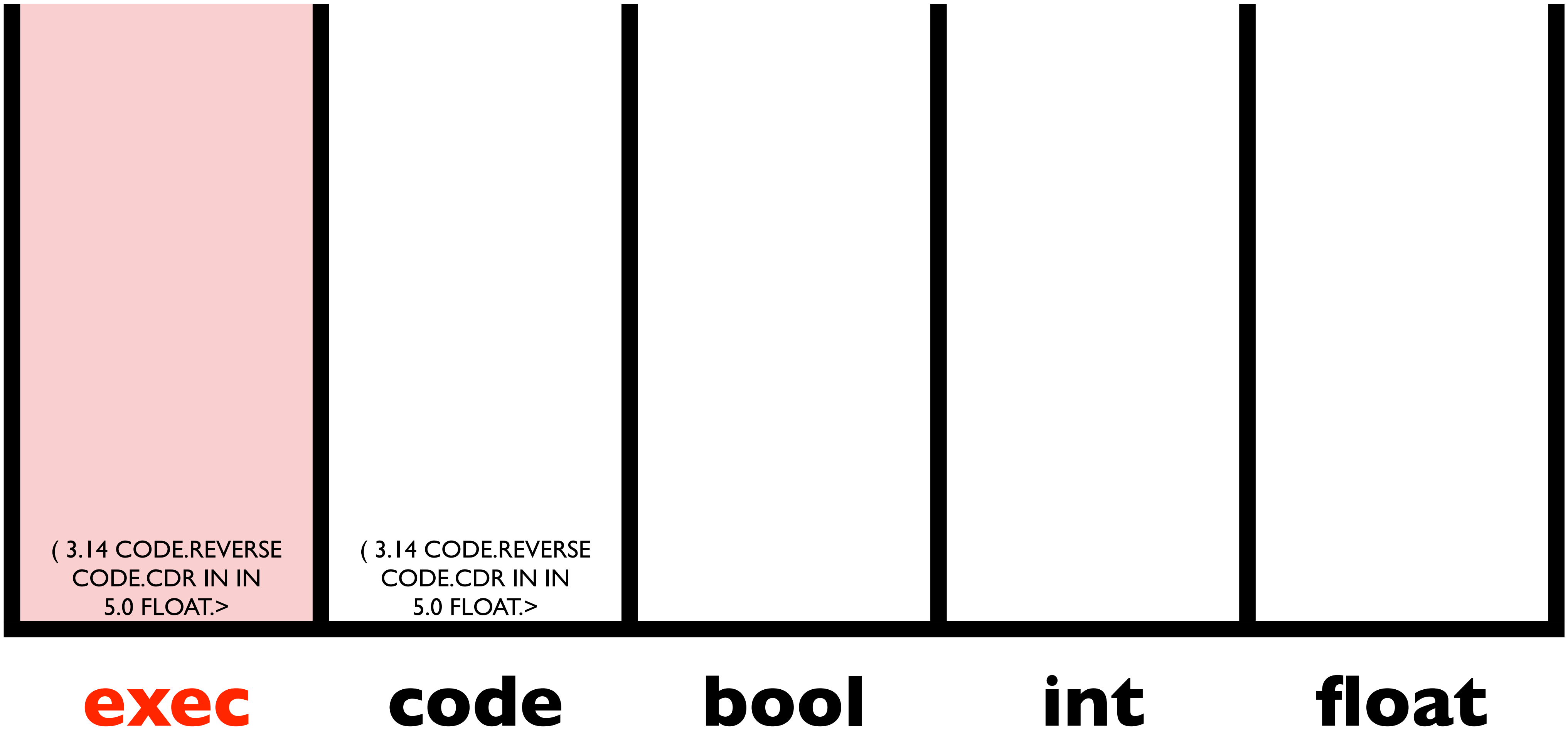
Same Results

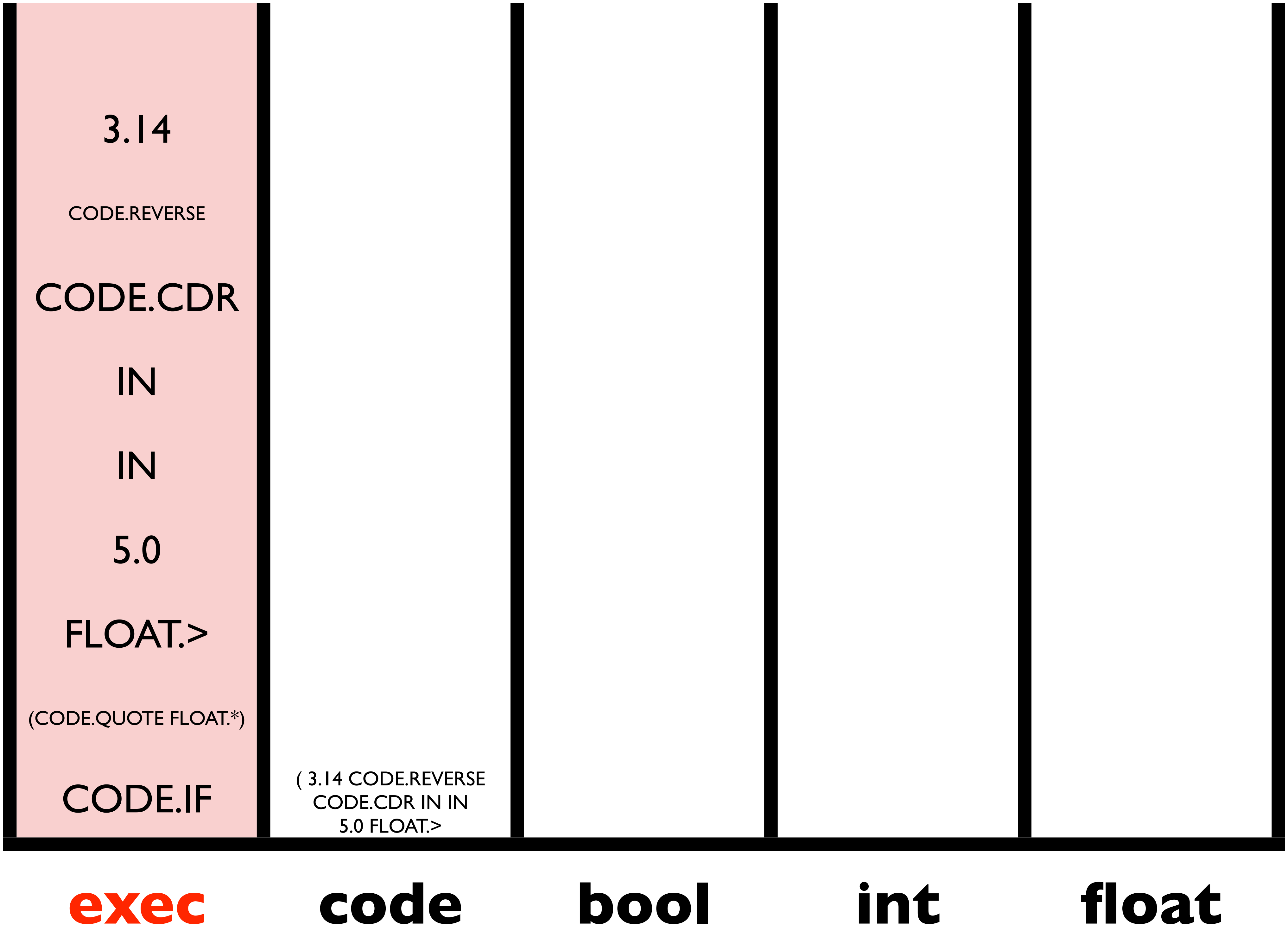
```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+
TRUE FALSE BOOLEAN.OR )
```

```
( 2 BOOLEAN.AND 4.1 TRUE INTEGER./ FALSE
3 5.2 BOOLEAN.OR INTEGER.* FLOAT.+ )
```

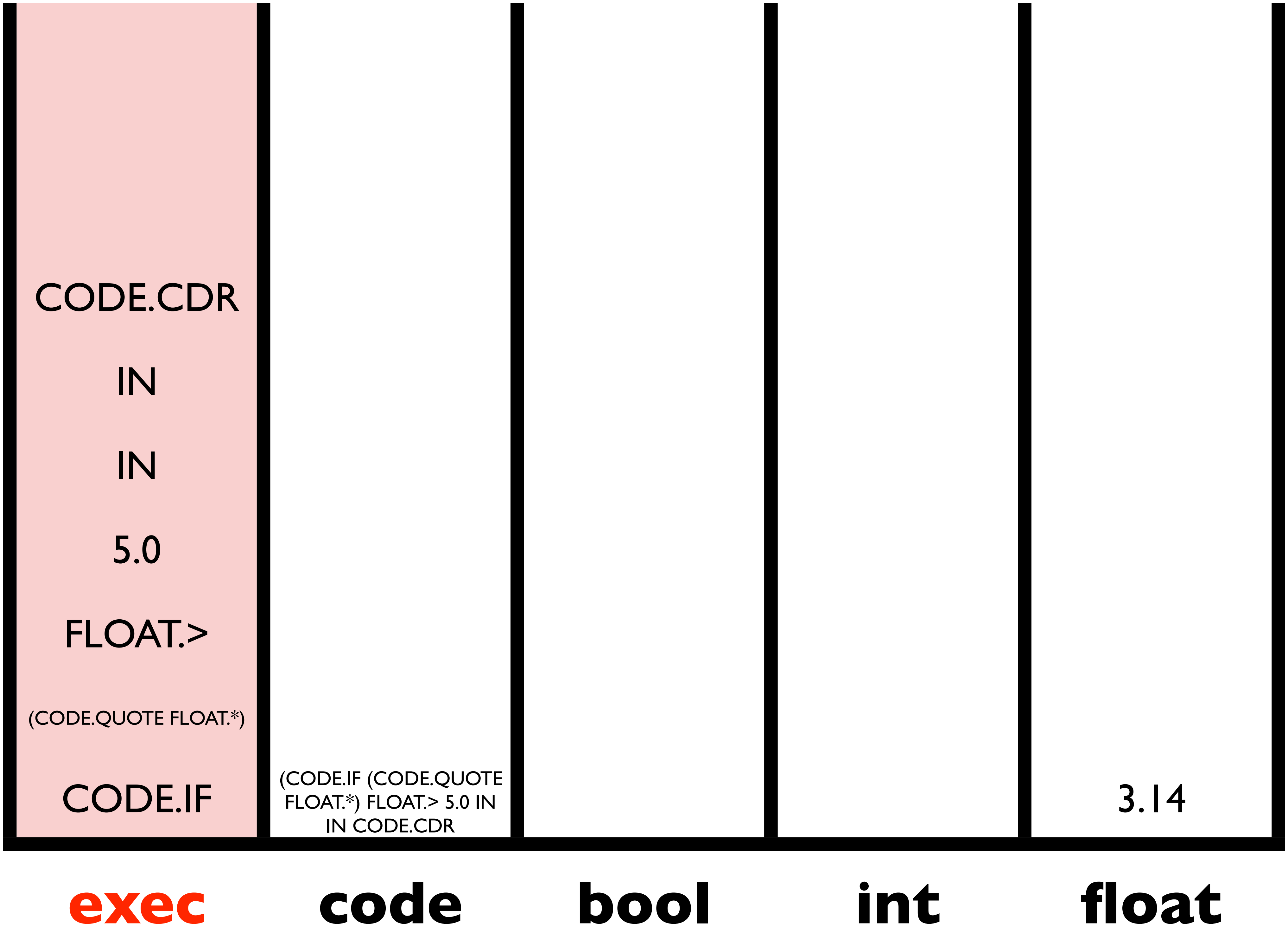
(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.> (CODE.QUOTE FLOAT.*) CODE.IF)

IN=4.0




3.14
CODE.REVERSE
CODE.CDR
IN
IN
5.0
FLOAT.>
(CODE.QUOTE FLOAT.*)
CODE.IF
(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.>
exec
code
bool
int
float

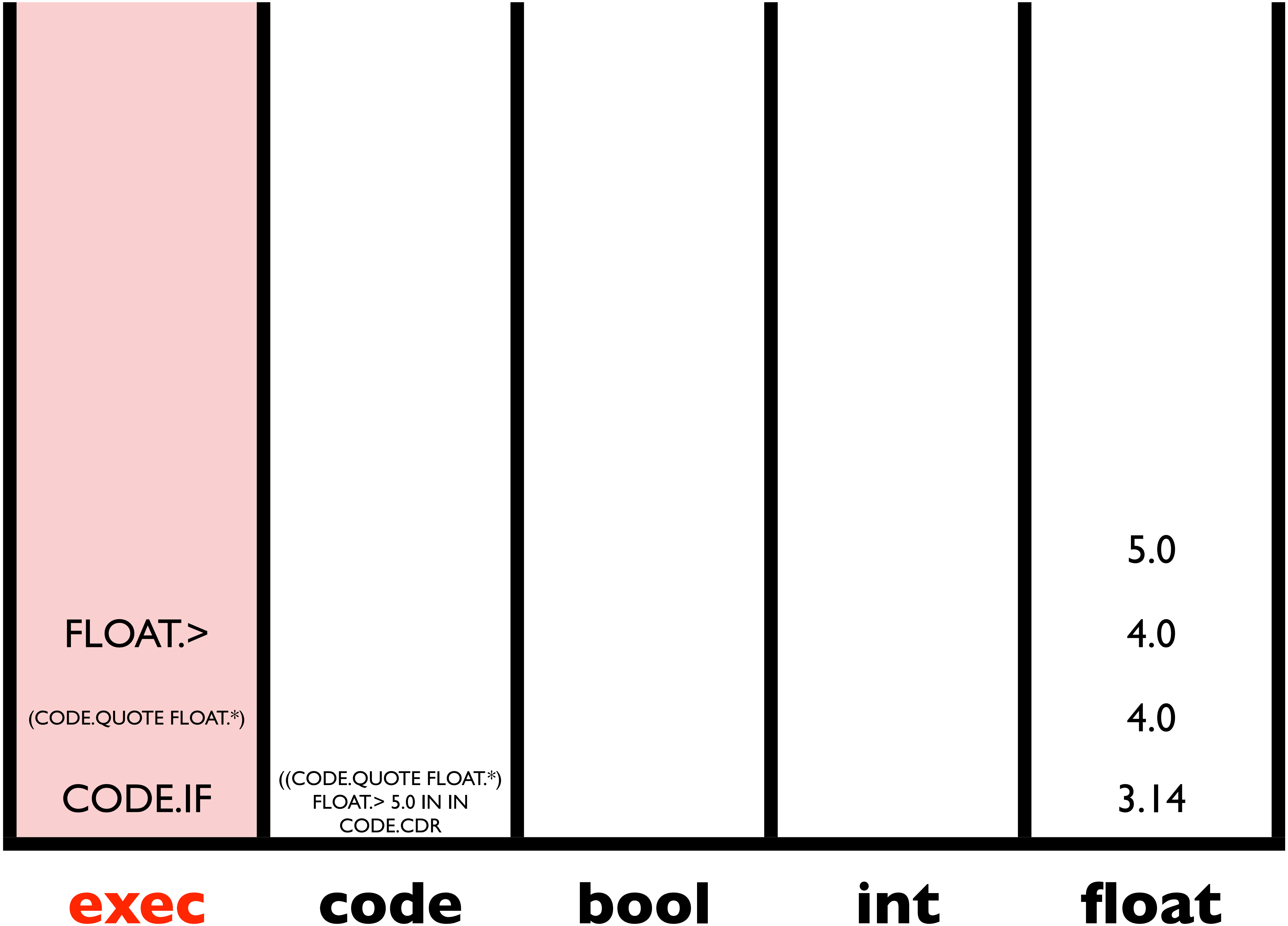
exec	code	bool	int	float
CODE.REVERSE				
CODE.CDR				
IN				
IN				
5.0				
FLOAT.>				
(CODE.QUOTE FLOAT.*)				
CODE.IF	(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.>			3.14

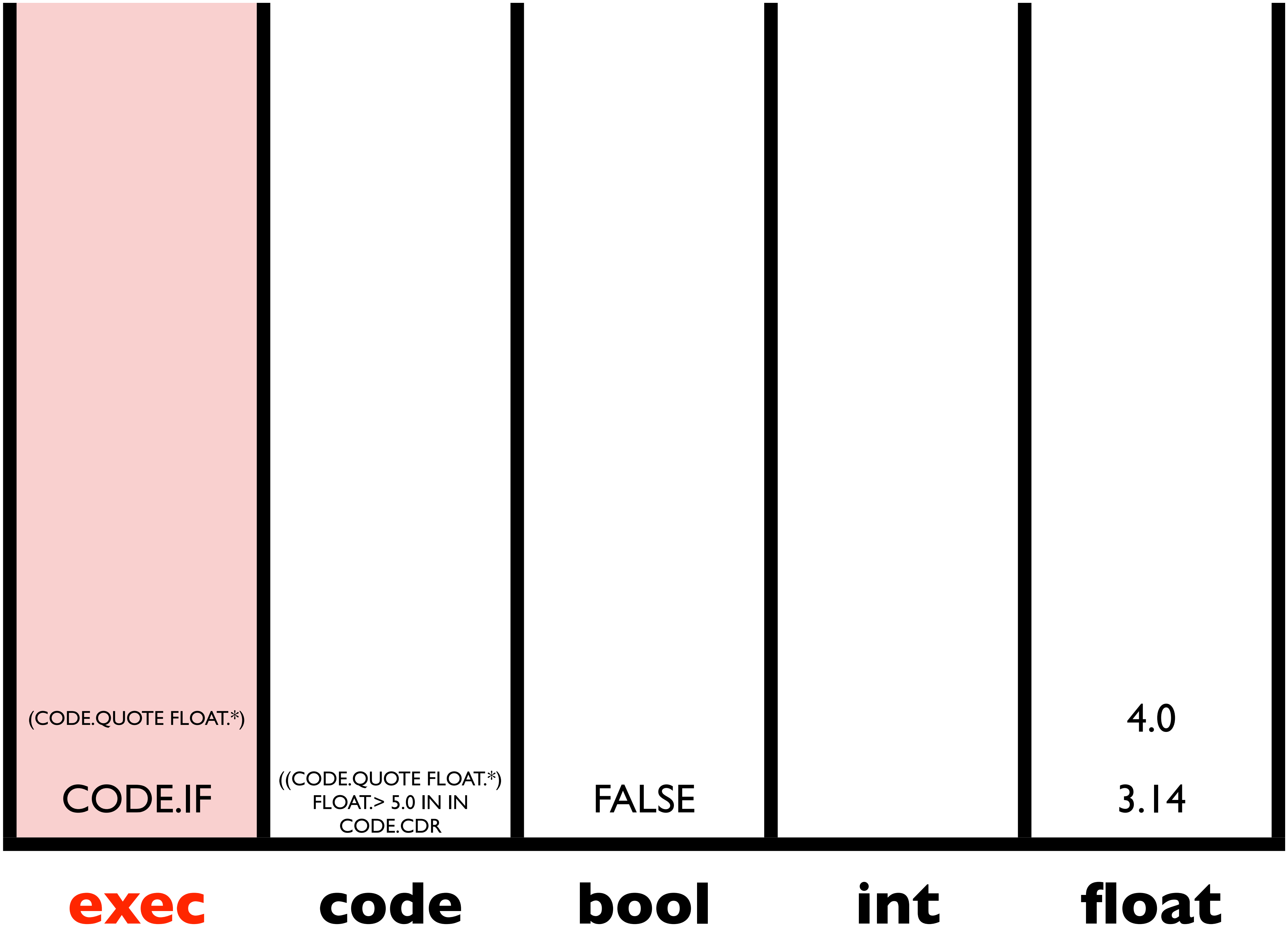

CODE.CDR
IN
IN
5.0
FLOAT.>
(CODE.QUOTE FLOAT.*)
CODE.IF
(CODE.IF (CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR
3.14
exec
code
bool
int
float

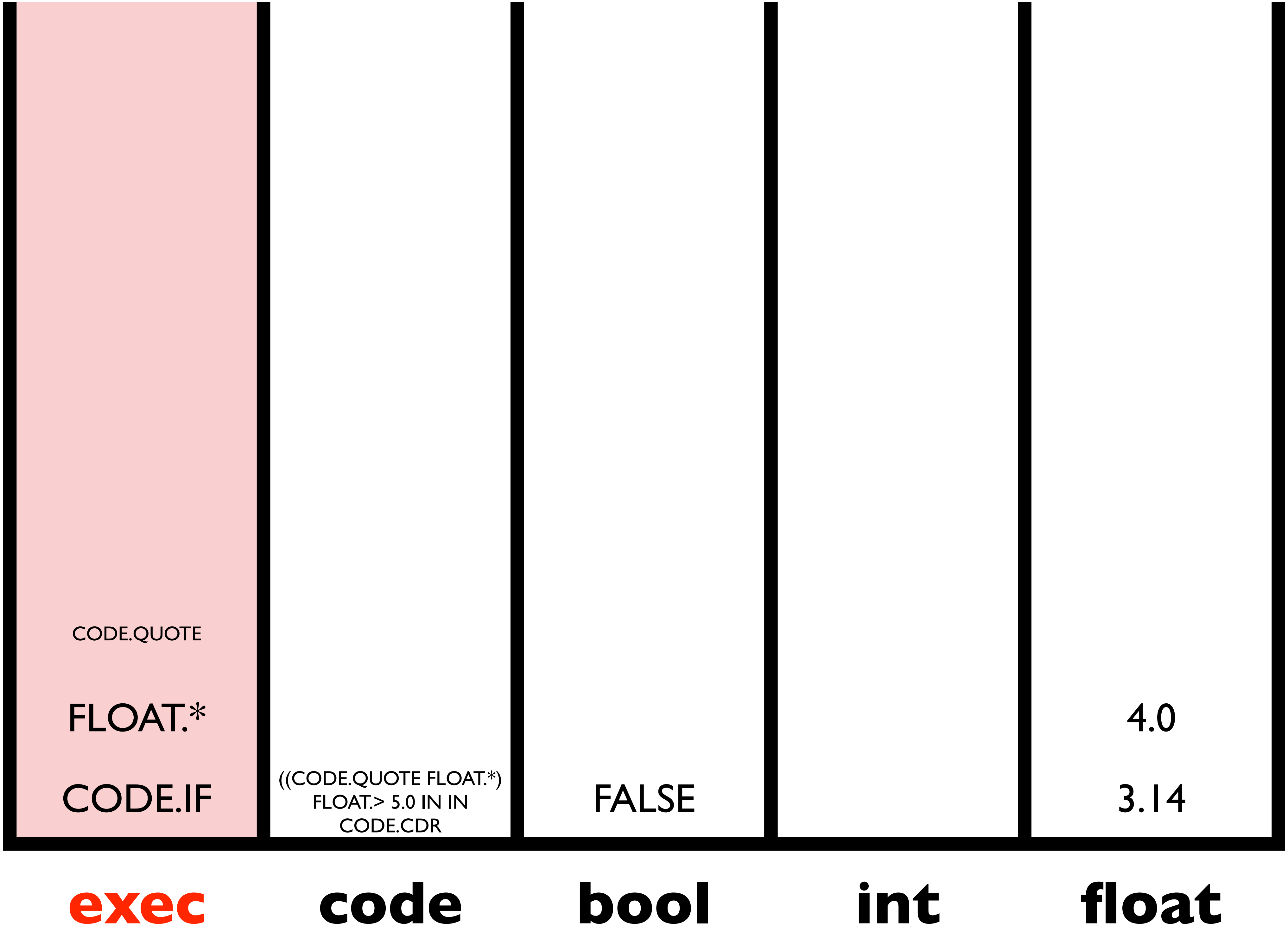
exec	code	bool	int	float
IN				
IN				
5.0				
FLOAT.>				
(CODE.QUOTE FLOAT.*)				
CODE.IF	((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR			3.14

IN				
5.0				
FLOAT.>				
(CODE.QUOTE FLOAT.*)				4.0
CODE.IF	((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR			3.14
exec	**code**	**bool**	**int**	**float**

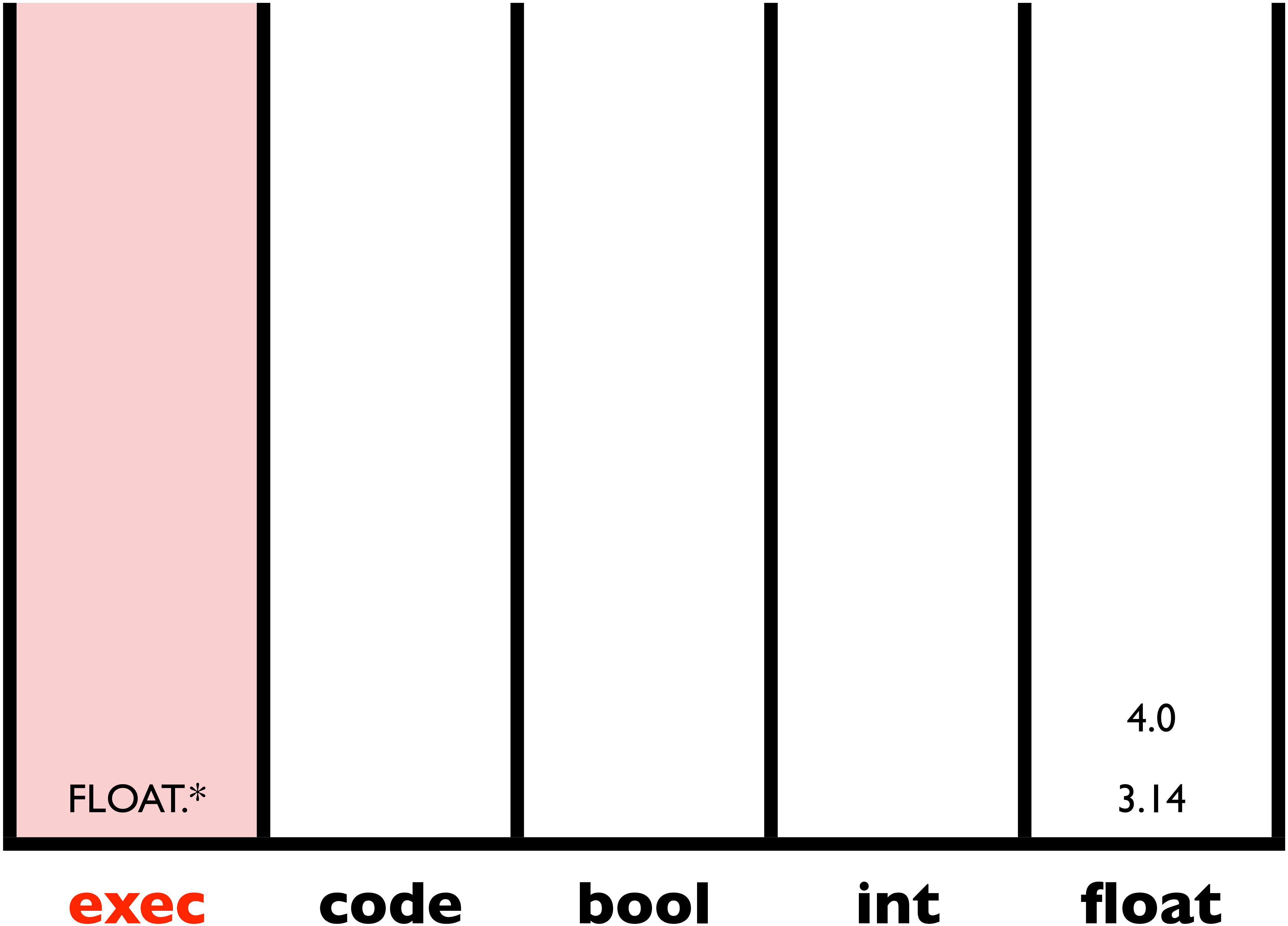
exec	code	bool	int	float
5.0				
FLOAT.>				4.0
(CODE.QUOTE FLOAT.*)				4.0
CODE.IF	((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR			3.14

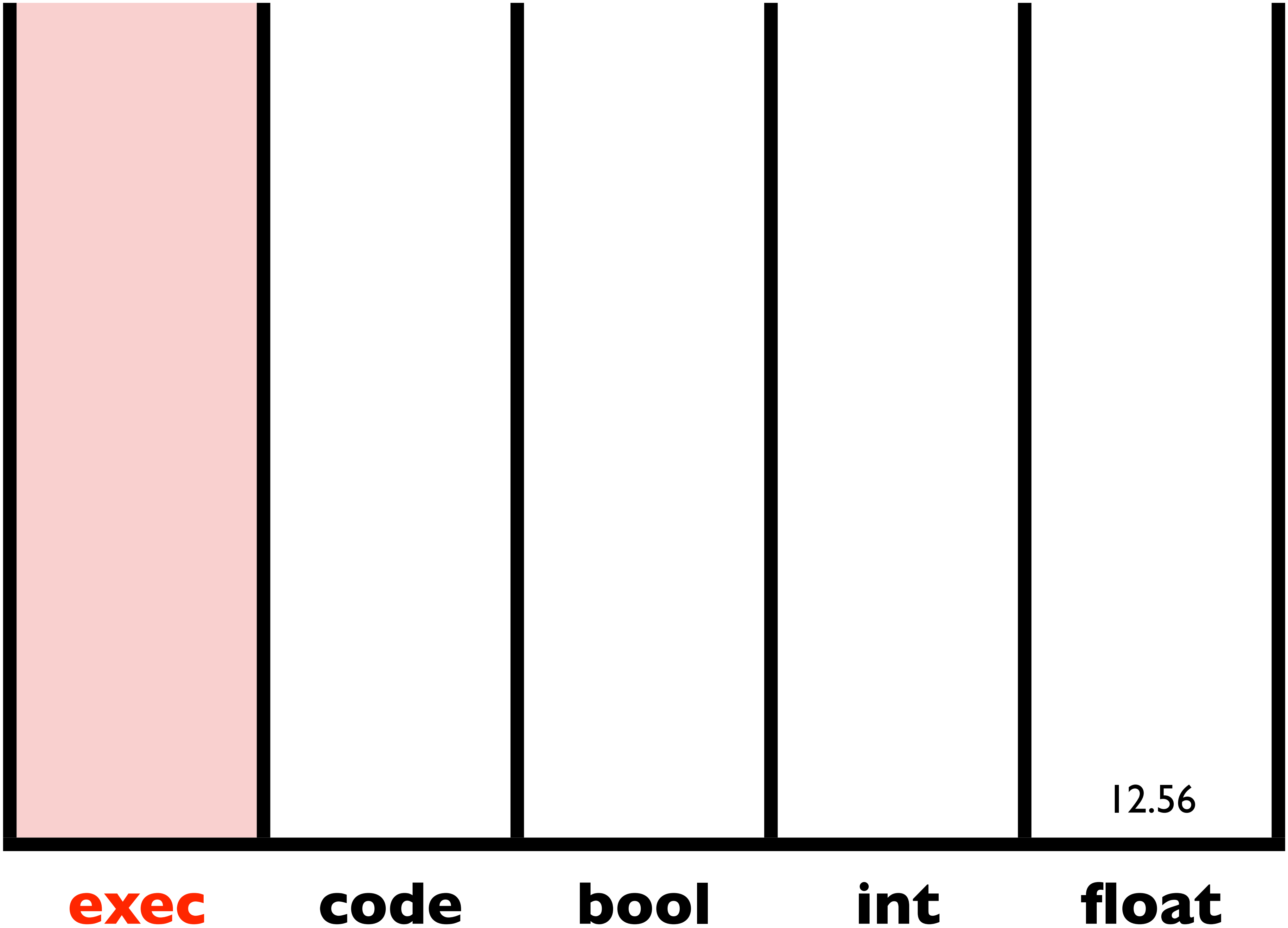

FLOAT.>
(CODE.QUOTE FLOAT.*)
CODE.IF
((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR
5.0
4.0
4.0
3.14
exec
code
bool
int
float


(CODE.QUOTE FLOAT.*)
CODE.IF
((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR
FALSE
4.0
3.14
exec
code
bool
int
float


CODE.QUOTE
FLOAT.*
CODE.IF
((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR
FALSE
4.0
3.14
exec
code
bool
int
float

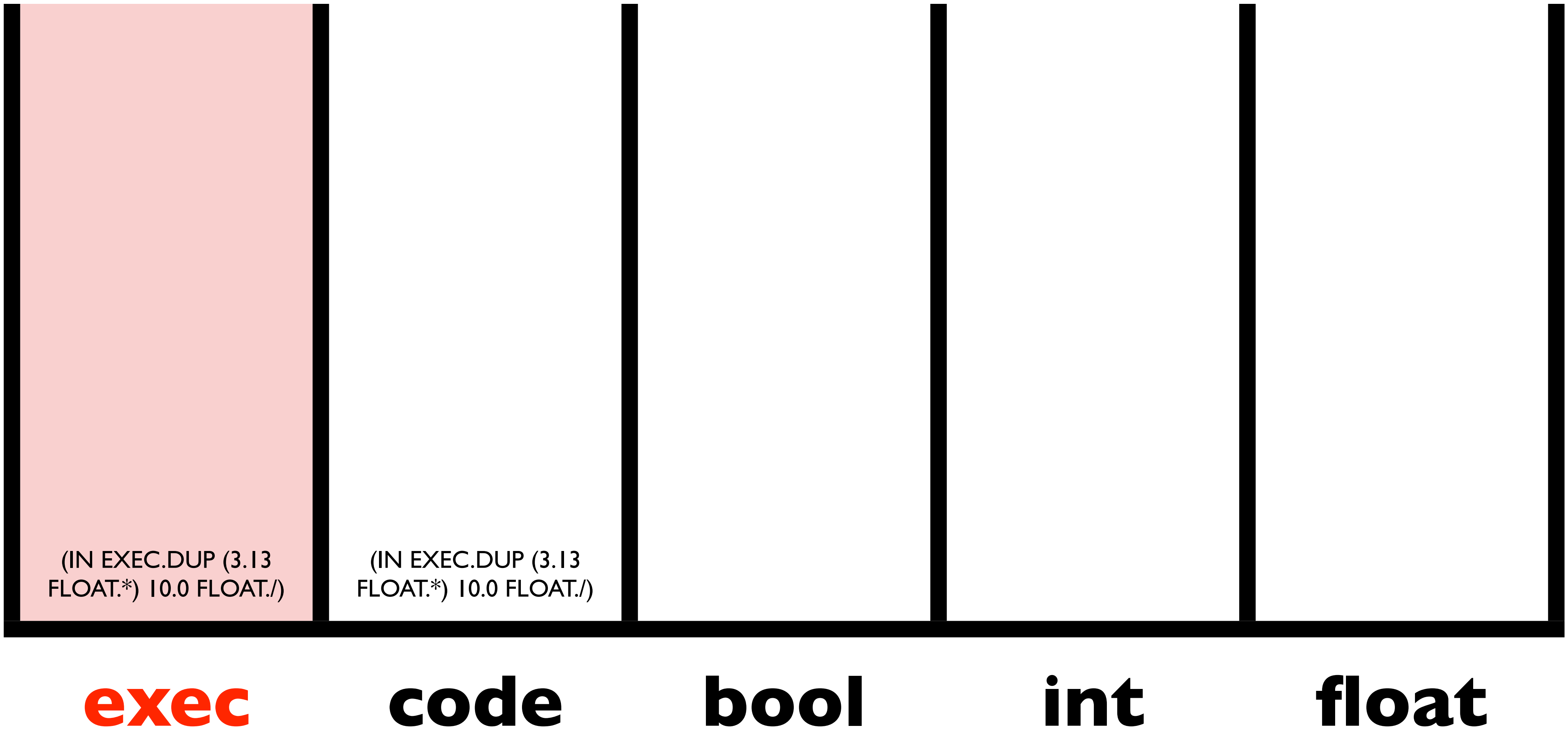
exec	code	bool	int	float
	FLOAT.*			4.0
CODE.IF	((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR	FALSE		3.14


FLOAT.*
4.0
3.14
exec
code
bool
int
float

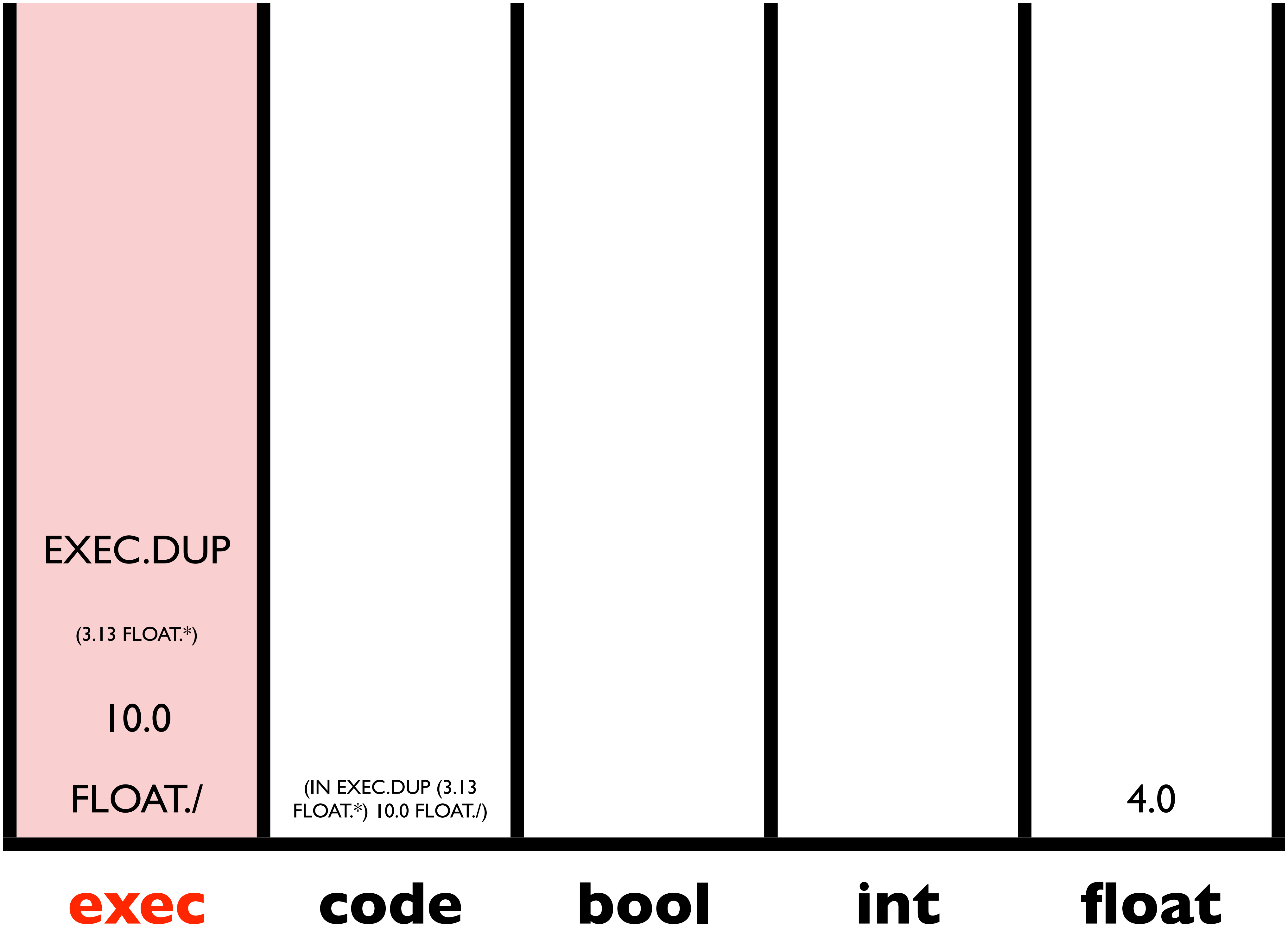

12.56
exec
code
bool
int
float

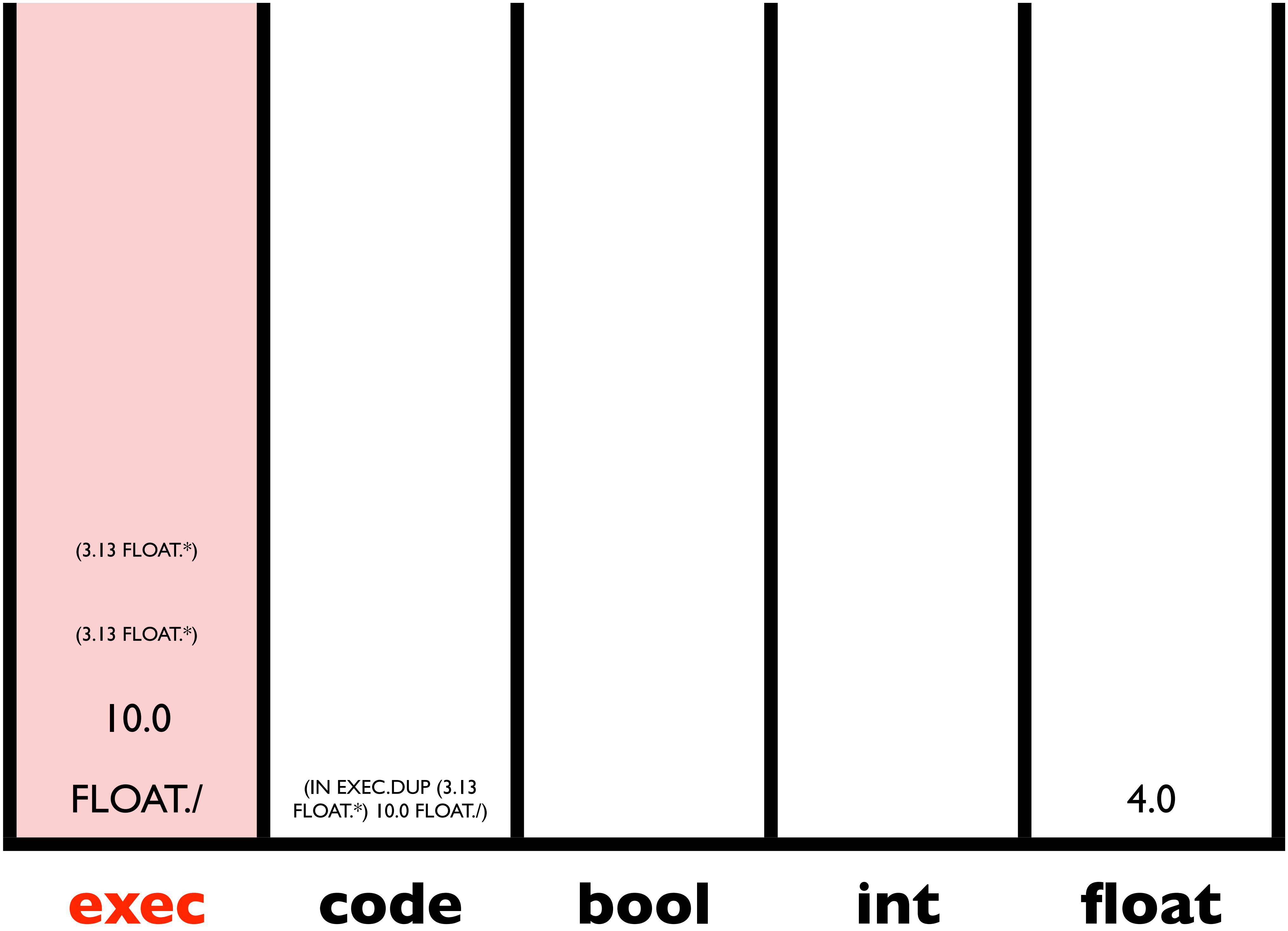
```
(IN EXEC.DUP (3.13 FLOAT.*)
10.0 FLOAT./)
```

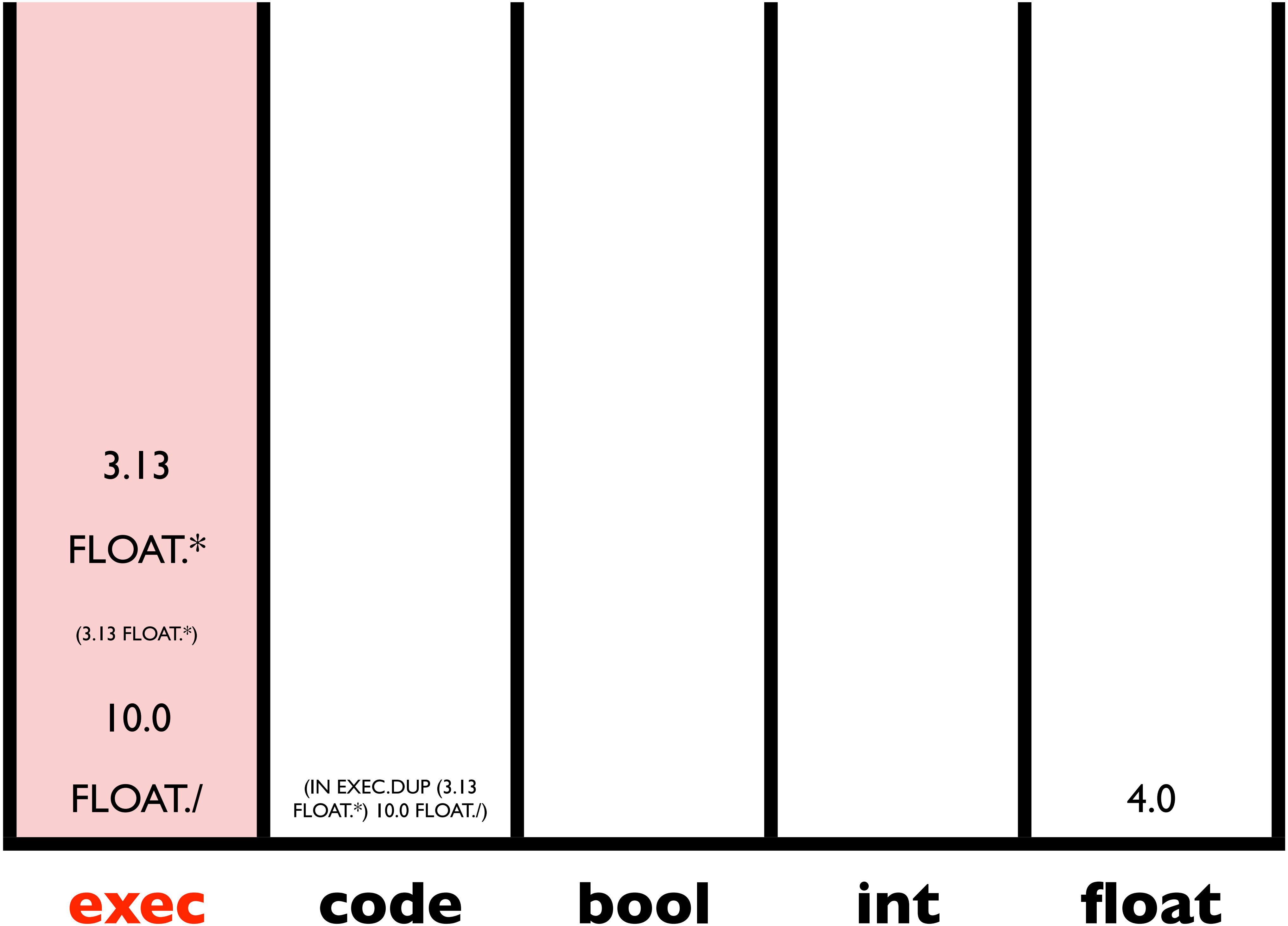
```
IN=4.0
```

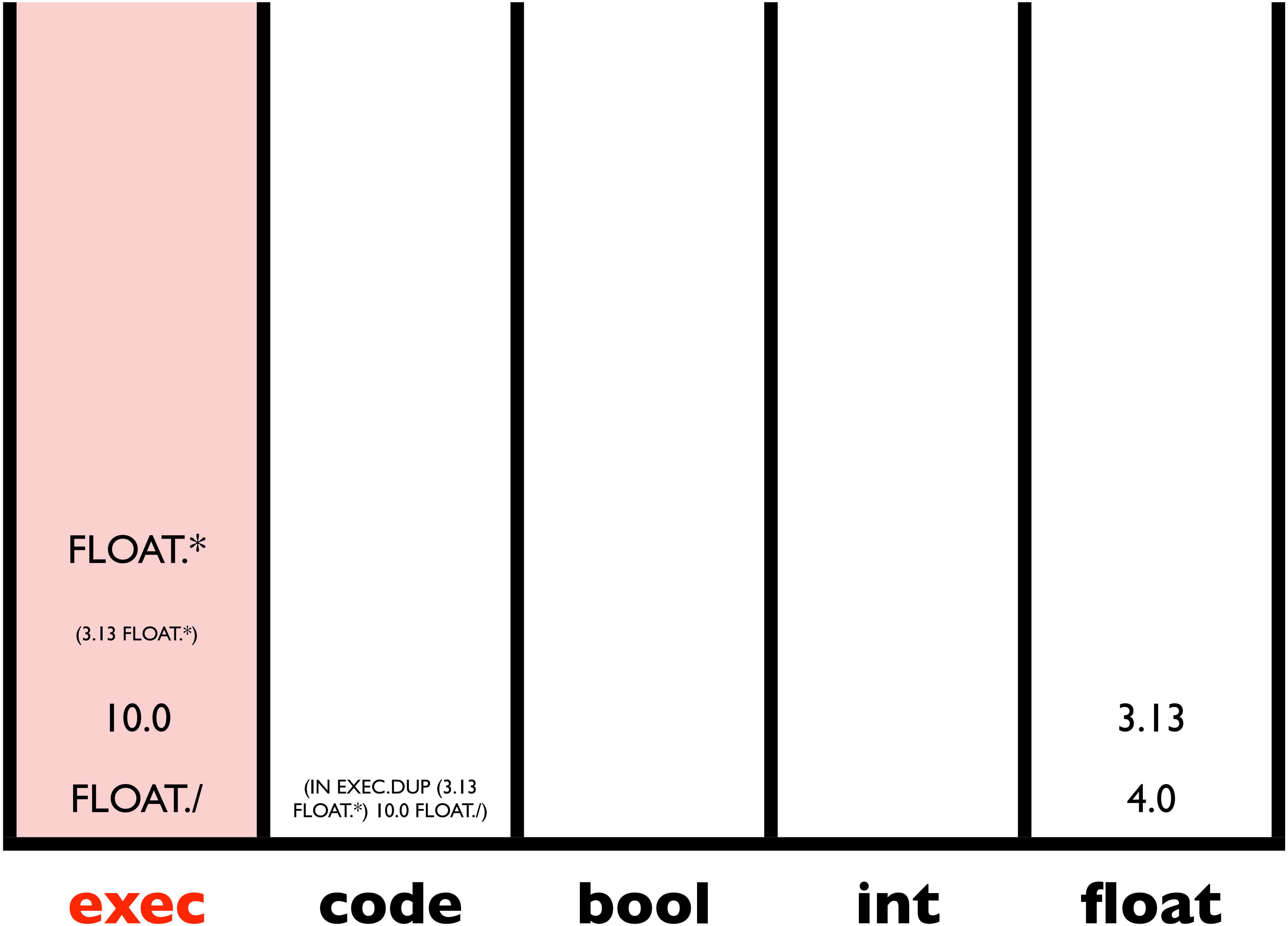


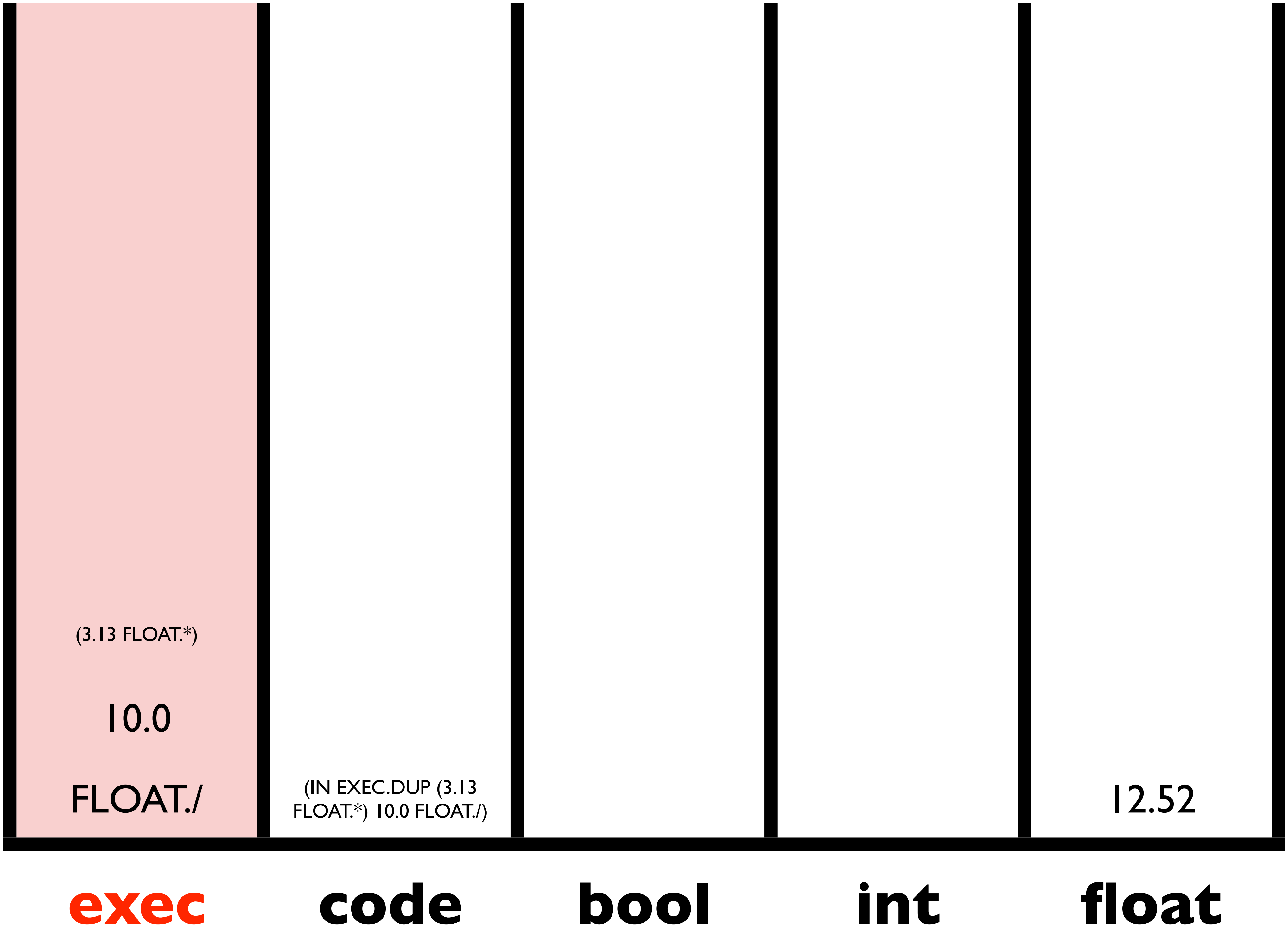
exec	code	bool	int	float
IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./	(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)			


EXEC.DUP
(3.13 FLOAT.*)
10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
4.0
exec
code
bool
int
float


(3.13 FLOAT.*)
(3.13 FLOAT.*)
10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
4.0
exec
code
bool
int
float

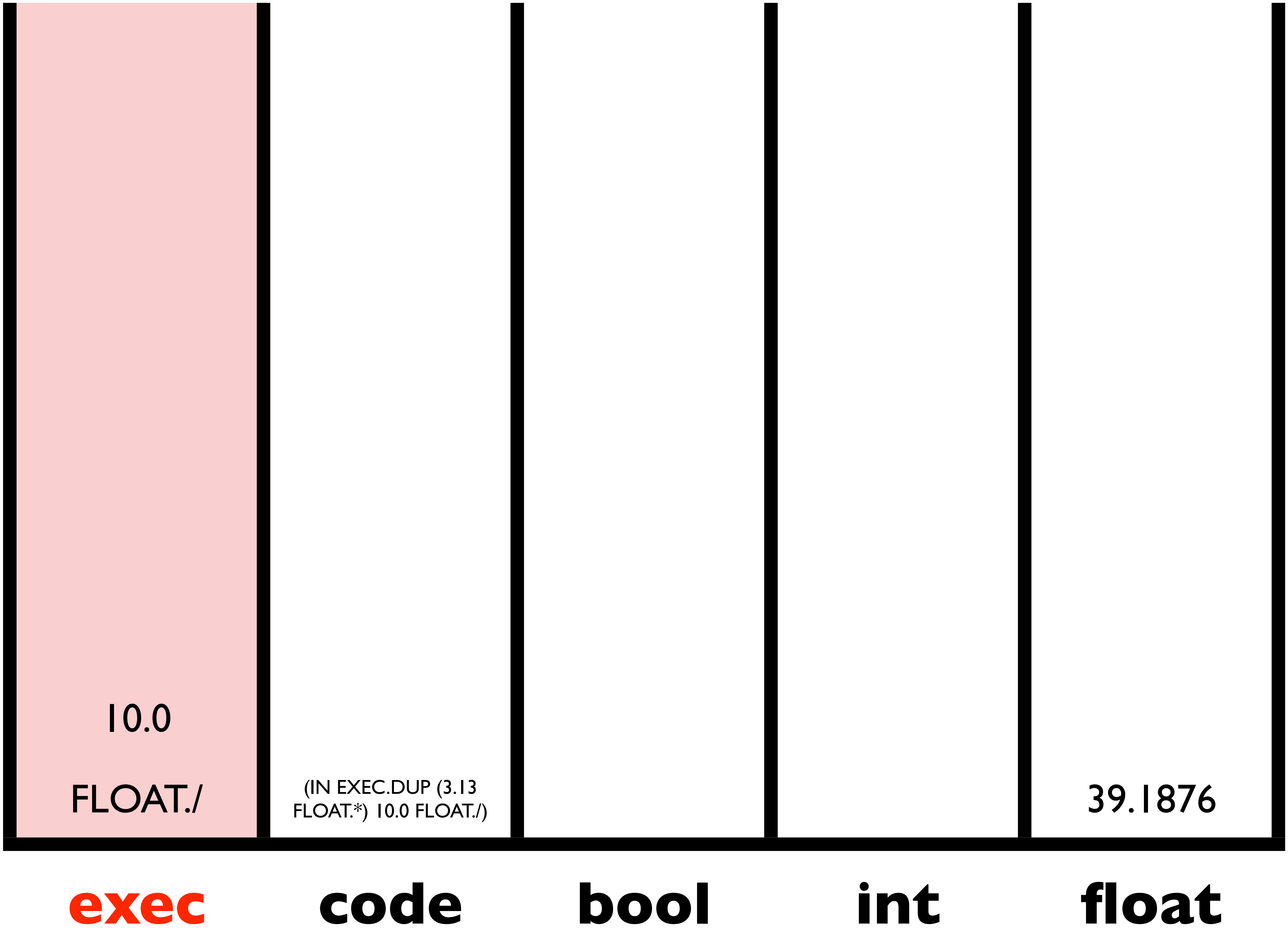

3.13
FLOAT.*
(3.13 FLOAT.*)
10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
4.0
exec
code
bool
int
float


FLOAT.*
(3.13 FLOAT.*)
10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
3.13
4.0
exec
code
bool
int
float

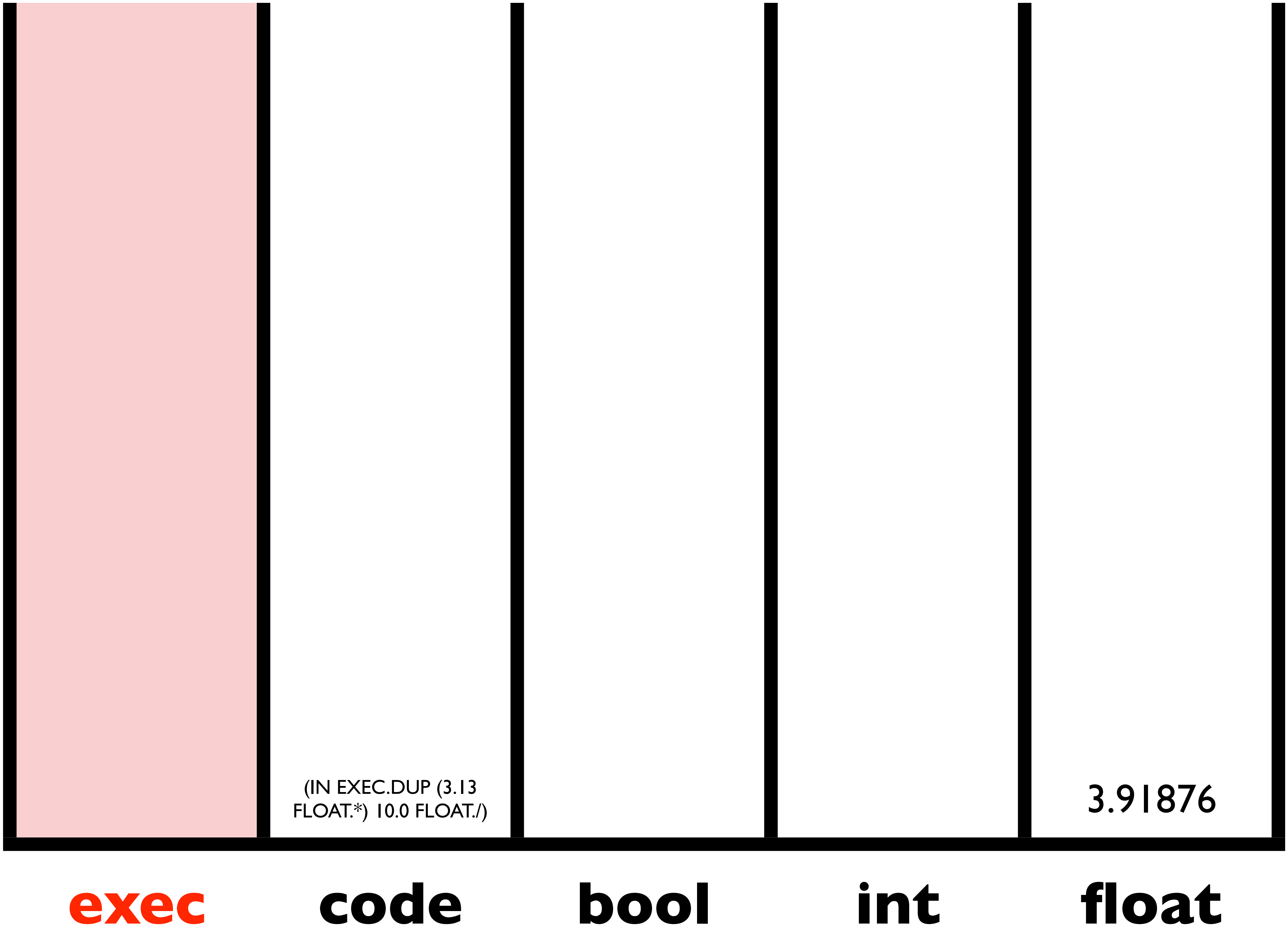

(3.13 FLOAT.*)
10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
12.52
exec
code
bool
int
float

exec	code	bool	int	float
3.13				
FLOAT.*				
10.0				
FLOAT./	(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)			12.52

exec	code	bool	int	float
FLOAT.*				
10.0				3.13
FLOAT./	(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)			12.52


10.0
FLOAT./
(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)
39.1876
exec
code
bool
int
float

				10.0
FLOAT./	(IN EXEC.DUP (3.13 FLOAT.*) 10.0 FLOAT./)			39.1876
exec	**code**	**bool**	**int**	**float**


(IN EXEC.DUP (3.13
FLOAT.*) 10.0 FLOAT./)
3.91876
exec
code
bool
int
float

Auto-simplification

Loop:

Make it randomly simpler

If it's as good or better: keep it

Otherwise: revert


STAPLES
SPL-110
12345678
%
OFF
ON/C
MRC
M-
M+
CE
7
8
9
÷
4
5
6
×
1
2
3
−
0
•
=
+

Calculator Test Cases

Keys pressed => number, error flag

- Digit entry tests
- Digit entry pair tests
- Double digit float entry tests
- Single digit math tests
- Single digit incomplete math tests
- Single digit chained math tests
- Division by zero tests

Digit Entry Tests

- :zero => 0.0, false
- :one => 1.0, false
- :two => 2.0, false
- :three => 3.0, false
- ...

Digit Entry Pair Tests

- :zero :zero => 0.0, false
- :zero :one => 1.0, false
- :two :three => 23.0, false
- :nine :nine => 99.0, false
- ...

Float Entry Tests

- :zero :point :nine => 0.9, false
- :zero :point :two => 0.2 false
- :seven :point :nine => 7.9, false
- :three :point :two => 3.2, false
- ...

Single Digit Math Tests

- :zero :plus :nine :equals => 9.0, false
- :three :times :four :equals => 12.0, false
- :three :minus :nine :equals => -6.0, false
- :three :divided-by :four :equals => 0.75, false
- ...

Incomplete Math Tests

- :three :plus :four => 4.0, false
- :seven :plus => 7.0, false
- ...

Chained Math Tests

- :three :plus :nine :minus :five :equals
  => 7.0, false
- :three :times :two :divided-by :eight :equals
  => 0.75, false
- :three :divided-by :nine :minus :five :equals
  => -4.6666665, false
- ...

Division by Zero Tests

- :zero :divided-by :zero :equals => 0.0, true
- :seven :divided-by :zero :equals => 0.0, true
- :three :divided-by :zero :equals => 0.0, true
- ...

Architectural Requirements

- Every key press is an entry point
- Answers (2) should provided after every key press
- State must be maintained between key presses
- Stacks + tags provide an elegant way to meet these requirements

Holland's Tags

- Initially arbitrary identifiers that come to have meaning over time
- Matches may be inexact
- Appear to be present in some form in many different kinds of complex adaptive systems
- Examples range from immune systems to armies on a battlefield
- A general tool for the support of emergent complexity

Tag-based Modules

- Include instructions that tag code (modules)
- Include instructions that recall and execute modules by *closest matching* tag
- If a single module has been tagged then all tag references will recall modules
- The number of tagged modules can grow incrementally over evolutionary time
- Expressive and evolvable

Tags in Push

- Tags are integers embedded in instruction names
- Instructions like `tag.exec.123` tag values
- Instructions like `tagged.456` recall values by *closest matching* tag
- If a single value has been tagged then all tag references will recall (and execute) values
- The number of tagged values can grow incrementally over evolutionary time

Calculator Architecture

- Run program once to tag modules
- Clear stacks
- For each pressed key, execute the module that best matches the corresponding tag, maintaining stacks across key presses
- The top of the float stack is the number output; the top of the boolean stack is the error flag output

And?

With Push and the tagged-entry-point architecture we can run GP on the calculator problem.

And it fails miserably:

- Different test cases require qualitatively different modes of response
- Numbers of cases of different types have an undue influence
- Average performance across cases does not guide search appropriately

Lexicase Selection

- Each parent is selected by filtering the entire population, one one case at a time (in random order), keeping only the elite at each stage
- Useful for "modal" problems, which require qualitatively different responses to different inputs
- Useful for "uncompromising" problems, in which solutions must be optimal on each case
- All comparisons are "within case," so may be useful whenever cases are non-comparable

Lexicase Selection

Initialize:

Candidates = the entire population

Cases = a list of all of the test cases in random order

Loop:

Candidates = the subset of **Candidates** with exactly the best performance of any current candidate for the first case in **Cases**

If **Candidates** or **Cases** contains just a single element then return a randomly selected individual from **Candidates**

Otherwise remove the first case from **Cases** and go to **Loop**

Finite Algebras

$\mathbf{A}_1$ $*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

$\mathbf{A}_2$ $*$	0	1	2
0	2	0	2
1	1	0	2
2	1	2	1

A1 Mal'cev Term

Selection	Successes	CE	MBF
Tournament Size 2	35	532,000	0.75
Tournament Size 3	43	420,000	0.70
Tournament Size 4	31	440,000	0.75
Tournament Size 5	22	616,000	0.77
Tournament Size 6	25	750,000	0.90
Tournament Size 7	23	403,000	0.92
Tournament Size 8	26	464,000	0.94
Tournament Size 9	21	550,000	1.06
Lexicase	94	90,000	0.05

A2 Mal'cev Term

Selection	Successes	CE	MBF
Tournament Size 3	7	3,780,000	1.50
Tournament Size 4	5	3,648,000	1.50
Tournament Size 5	8	2,052,000	1.51
Tournament Size 6	9	1,921,000	1.45
Tournament Size 7	3	4,131,000	1.59
Tournament Size 8	9	990,000	1.64
Tournament Size 9	10	1,356,000	1.60
Lexicase	75	208,000	0.25

Digital Multiplier

- Evolve a digital circuit to multiply two binary numbers
- *n*-bit digital multiplier: 2 x *n* bits → 2*n* bits
- Multiple outputs
- Scalable
- Recommended as a GP benchmark problem (McDermott, et al 2012, White et al 2013)

3-bit Digital Multiplier

Boolean Stack	and, or, xor, invert_first_then_and, dup, swap, rot
Input / Output	in0, ..., in2n, out0, ..., out2n

Selection	**Successes**	**MBF**
Tournament Size 7	0	0.24
Lexicase	100	0

Factorial

Boolean Stack	and, dup, eq, frominteger, not, or, pop, rot, swap
Integer Stack	add, div, dup, eq, fromBoolean, greaterThan, lessThan, mod, mult, pop, rot, sub, swap
Exec Stack	dup, eq, if , noop, pop, rot, swap, when, k, s, y
Input	in
Constants	0, 1

Selection	**Successes**	**MBF**
Tournament Size 7	0	74,545
Lexicase	61	28,980

And?

With Push, the tagged-entry-point architecture, and lexicase selection... we still fail.

But not quite as miserably.

Issues:

- Large programs are required
- Must allow growth without bloating
- Must allow arbitrary recombination

Uniform Variation

- All genetic material that a child inherits should be ≈ likely to be mutated
- Parts of both parents should be ≈ likely to appear in children (at least if they are ≈ in size), and to appear in a range of combinations

Why Uniformity?

- No hiding from mutation
- All parts of parents subject to variation and recombination
- Biological genetic variation, while not fully uniform, has uniformity properties that prevent some of the problems we see in GP; e.g. just having more genes doesn't generally "protect" genes any of them

Prior Work

- Point mutations or "uniform crossovers" that replace/swap nodes but only in restricted ways; cannot change structure, has depth biases (McKay et al, 1995; Page et al, 1998; Poli and Langdon, 1998; Poli and Page, 2000; Semenkin and Semenkina, 2012)
- Uniform mutation via size-based numbers of tree replacements; depth biases, little demonstrated benefit (McKay et al, 1995; Van Belle and Ackley, 2002)

ULTRA

- Achieve uniformity by treating genomes as linear sequences, even if they are hierarchically structured
- Repair after transform to ensure structural validity

The ULTRA Operator

- **U**niform **L**inear **T**ransformation with **R**epair and **A**lternation
- Linearize 2 parents, treating "(" and ")" as ordinary tokens
- Start at the beginning of one parent and copy tokens to the child, switching parents stochastically (according to the *alternation rate*, and subject to an *alignment deviation*)
- Post-process with uniform mutation (according to a *mutation rate*) and repair

Parents:

(a b (c (d)) e (f g))

(1 (2 (3 4) 5) 6)

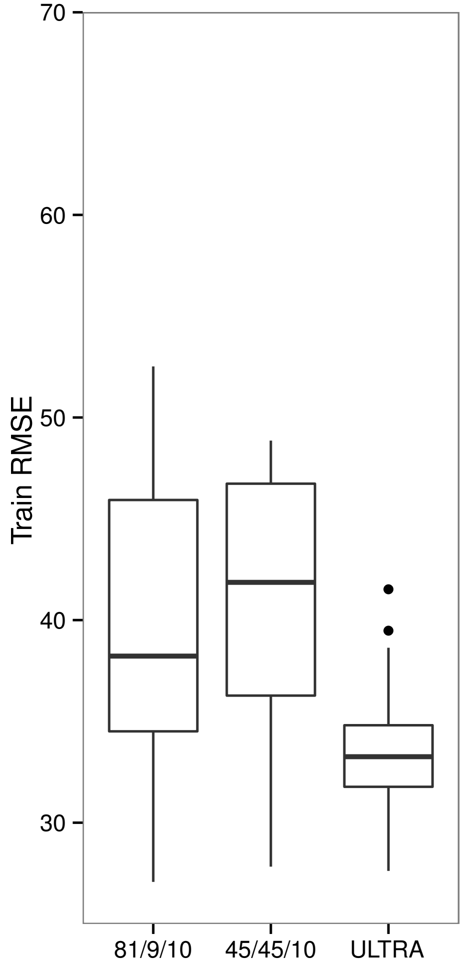
Result of alternation:

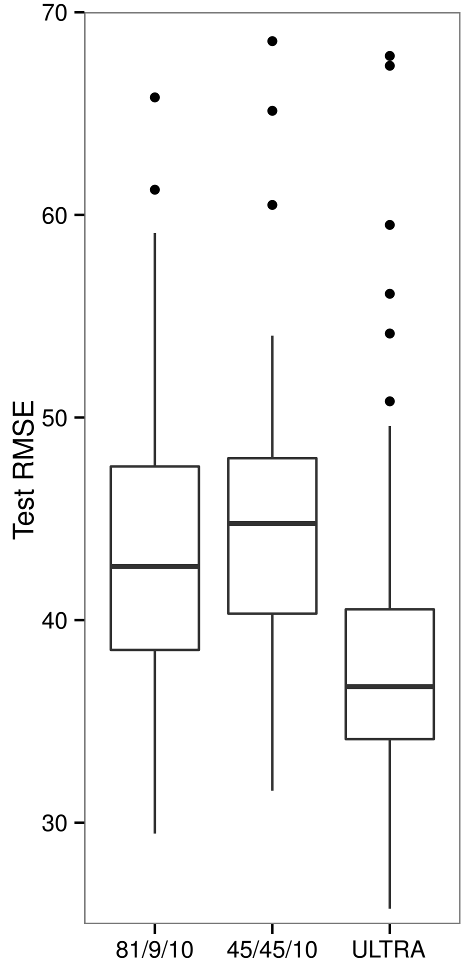
(a b 2 (3 4 d)) 6)

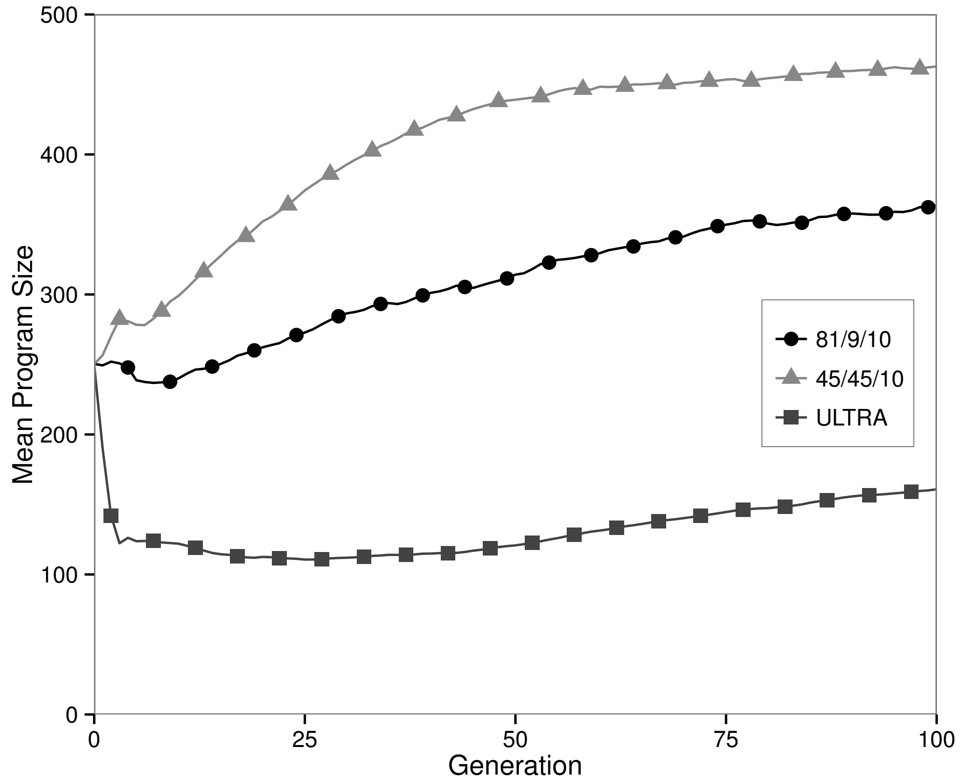
Result of repair:

(a (b 2 (3 4 d)) 6)

ULTRA on the bioavailability problem







And?

With Push, the tagged-entry-point architecture, lexicase selection, and ULTRA... we succeed!

On some reasonably large sets of tests (not all shown above).

But without generalizing.

Continuing Work

- Generative tests for selection and validation
- Refinements to lexicase selection, ULTRA, and tagging mechanisms
- Parallel work on other program synthesis problems:
  - Kata bowling
  - The UNIX wc program
- Synergize with non-evolutionary program synthesis work

Conclusions

- Evolutionary synthesis of arbitrary software is hard!
- But we can learn a lot trying to do it, both for software synthesis and for other GP applications
- Push, tags, tagged-entry points, uniform variation methods, and lexicase selection have all demonstrated promise

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