Expressive Genetic Programming

Tutorial

Genetic and Evolutionary Computation Conference (GECCO-2015) Madrid, Spain

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Instructor



Lee Spector is a Professor of Computer Science in the School of Cognitive Science at Hampshire College in Amherst, Massachusetts, and an adjunct professor in the Department of Computer Science at the University of Massachusetts, Amherst. He received a B.A. in Philosophy from Oberlin College in 1984 and a Ph.D. from the Department of Computer Science at the University of Maryland in 1992. His areas of teaching and research include genetic and evolutionary computation, quantum computation, and a variety of intersections between computer science, cognitive science, evolutionary biology, and the arts. He is the Editor-in-Chief of the journal Genetic Programming and Evolvable Machines (published by Springer) and a member of the editorial board of Evolutionary Computation (published by MIT Press). He is also a member of the SIGEVO executive committee and he was named a Fellow of the International Society for Genetic and Evolutionary Computation.

Tutorial Description (1)

The language in which evolving programs are expressed can have significant impacts on the problem-solving capabilities of a genetic programming system. These impacts stem both from the absolute computational power of the languages that are used, as elucidated by formal language theory, and from the ease with which various computational structures can be produced by random code generation and by the action of genetic operators. Highly expressive languages can facilitate the evolution of programs for any computable function using, when appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

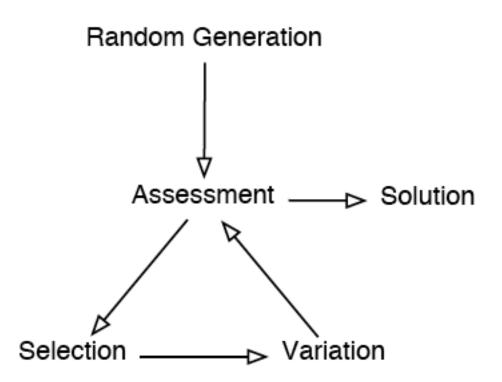
Tutorial Description (2)

This tutorial will begin by presenting a range of approaches that have been taken for evolving programs in expressive programming languages. Within this context it will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness and specifically for use in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, supporting the evolution of complex, modular programs in a particularly simple and flexible way. The Push language will be described and demonstrated, and over ten years of Push-based research, including the production of human-competitive results, will be briefly surveyed. The tutorial will conclude with a discussion of recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

Course Agenda

- Genetic Programming
- Evolving programs in expressive languages
- Expressivity, evolvability, and syntactic minimality
- Push + DEMO
- Expressing the evolution of expressive evolution

Evolutionary Computation



Evolution, the Designer

"Darwinian evolution is itself a designer worthy of significant respect, if not religious devotion." *Boston Globe* OpEd, Aug 29, 2005

WHAT WOULD DARWIN SAY? | LEE SPECTOR

The Boston Globe

And now, digital evolution

By Lee Spector | August 29, 2005

RECENT developments in computer science provide new perspective on "intelligent design," the view that life's complexity could only have arisen through the hand of an intelligent designer. These developments show that complex and useful designs can indeed emerge from random Darwinian processes.

Genetic Programming (GP)

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software

Program Representations

- Lisp-style symbolic expressions (Koza, ...).
- Purely functional/lambda expressions (Walsh, Yu, ...).
- Linear sequences of machine/byte code (Nordin et al., ...).
- Artificial assembly-like languages (Ray, Adami, ...).
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...).
- Graph-structured programs (Teller, Globus, ...).
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...)
- Fuzzy rule systems (Tunstel, Jamshidi, ...)
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...).
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, ...).

Mutating Lisp

```
(+ (* X Y)

(+ 4 (- Z 23)))

(+ (* X Y)

(+ 4 (- Z 23)))

(+ (- (+ 2 2) Z)

(+ 4 (- Z 23)))
```

Recombining Lisp

```
Parent 1: (+ (* X Y)
           (+ 4 (- Z 23)))
Parent 2: (- (* 17 (+ 2 X))
          (* (- (* 2 Z) 1)
              (+ 14 (/ Y X)))
Child 1: (+ (- (* 2 Z) 1)
          (+ 4 (- Z 23)))
Child 2: (- (* 17 (+ 2 X))
          (* (* X Y)
              (+ 14 (/ Y X)))
```

Symbolic Regression

A simple example

Given a set of data points, evolve a program that produces y from x.

Primordial ooze: +, -, *, %, x, 0.1

Fitness = error (smaller is better)

GP Parameters

Maximum number of Generations: 51

Size of Population: 1000

Maximum depth of new individuals: 6

Maximum depth of new subtrees for mutants: 4

Maximum depth of individuals after crossover: 17

Fitness-proportionate reproduction fraction: 0.1

Crossover at any point fraction: 0.3

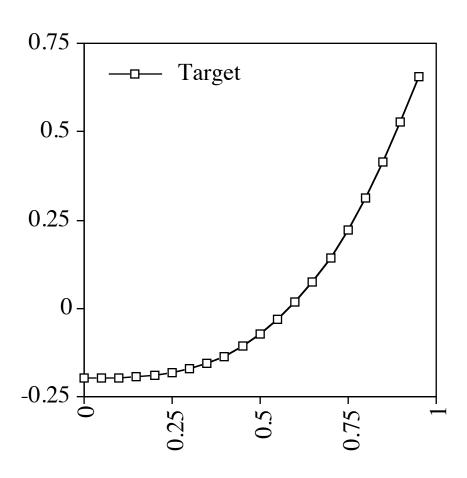
Crossover at function points fraction: 0.5

Selection method: FITNESS-PROPORTIONATE

Generation method: RAMPED-HALF-AND-HALF

Randomizer seed: 1.2

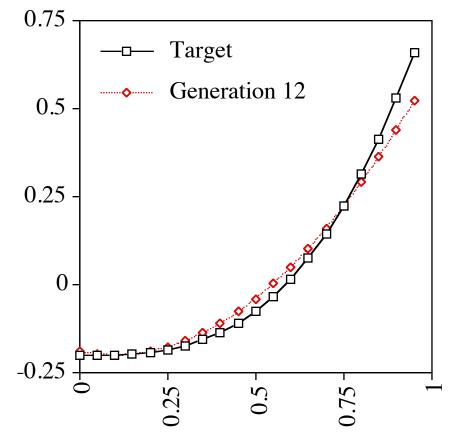
Evolving $y = x^3-0.2$



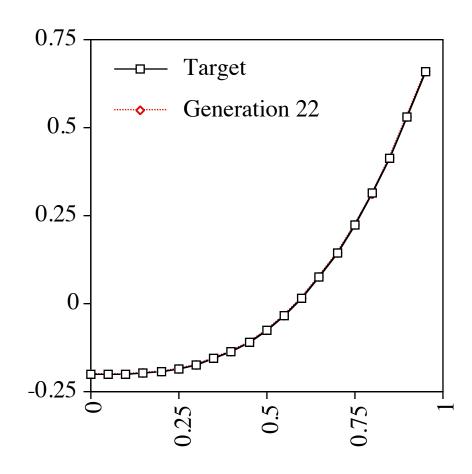
```
Target
                         0.75
                               Generation 0
(- (% (* 0.1
         (* X X))
                          0.5 -
       (- (% 0.1 0.1)
          (* X X)))
                         0.25
   0.1)
```

```
0.75
                                 Target
                              Generation 5
                          0.5
(- (* (* (% X 0.1)
          (* 0.1 X))
                         0.25
          (% 0.1 X)))
   0.1)
                           0
```

```
(+ (- (- 0.1)
         (-0.1)
            (-(*XX)
               (+ 0.1
                  (-0.1)
                     (* 0.1
                         0.1)))))
      (* X
         (* (% 0.1
               (% (* (* (- 0.1 0.1)
                         ( + X
                            (-0.10.1))
                     X)
                  (+ X (+ (- X 0.1)
                           (* X X)))))
            (+ 0.1 (+ 0.1 X))))
   (* X X))
```



```
(- (- (* X (* X X))
0.1)
0.1)
```



Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Partial recursive functions
- Register machine programs
- Assembly language programs
- etc.

Evolvability

The fact that a computation *can* be expressed in a formalism does not imply that a correct expression can be produced in that formalism by a human programmer or by an evolutionary process.

Data/Control Structure

Data abstraction and organization

Data types, variables, name spaces, data structures, ...

Control abstraction and organization

Conditionals, loops, modules, threads, ...

Structure via GP (1)

- Specialize GP techniques to directly support human programming language abstractions
- Strongly typed genetic programming
- Module acquisition/encapsulation systems
- Automatically defined functions
- Automatically defined macros
- Architecture altering operations

Evolving Modular Programs

With "automatically defined functions"

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

ADMs

- Macros implement control structures
- ADMs can be implemented via small tweaks to any system that supports ADFs
- Similar pros and cons to ADFs, but provide additional expressive power

Control Structures (1)

```
Multiple evaluation

(defmacro do-twice (code)

(progn ,code ,code))

(do-twice (incf x))
```

Control Structures (2)

Conditional evaluation

```
(defmacro numeric-if (exp neg zero pos)
  `(if (< ,exp 0)
  ,neg
  (if (< 0 ,exp) ,pos ,zero)))
(numeric-if (foo) (bar) (baz) (bix))</pre>
```

Structure via GP (2)

- Specialize GP techniques to **indirectly** support human programming language abstractions
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

Structure via GP (3)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- Minimal syntax + maximal semantics
- Push

Push

- Designed for program evolution
- Data flows via stacks, not syntax
- One stack per type: integer, float, boolean, string, code, exec, vector, ...
- Minimal syntax:
 program → instruction | literal | (program*)
- Missing argument? NOOP

Why Push?

- Highly expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Extensible
- Supports uniform variation
- Supports several forms of meta-evolution

Plush

Instruction	integer_eq	exec_dup	char_swap	integer_add	exec_if
Close?	2	0	0	0	1
Silence?	1	0	0	1	0

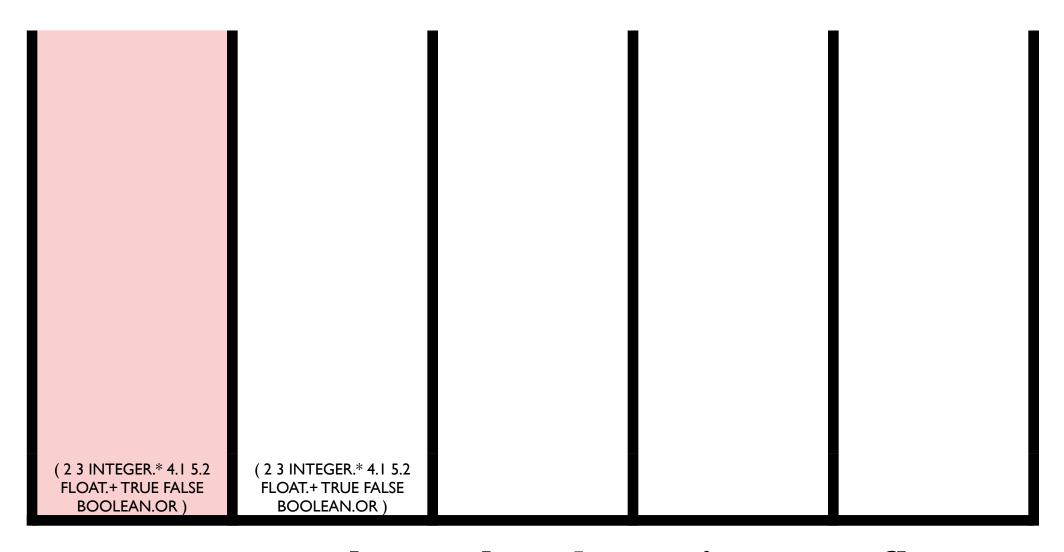
Sample Push Instructions

Stack manipulation	POP, SWAP, YANK,
instructions	DUP, STACKDEPTH,
(all types)	$\mathtt{SHOVE},\ \mathtt{FLUSH},=$
Math	+, -, /, *, >, <,
(INTEGER and FLOAT)	MIN, MAX
Logic (BOOLEAN)	AND, OR, NOT,
	FROMINTEGER
Code manipulation	QUOTE, CAR, CDR, CONS,
(CODE)	INSERT, LENGTH, LIST,
	MEMBER, NTH, EXTRACT
Control manipulation	DO*, DO*COUNT, DO*RANGE,
(CODE and EXEC)	DO*TIMES, IF

Push(3) Semantics

- To execute program P:
 - 1. Push P onto the EXEC stack.
 - 2. While the EXEC stack is not empty, pop and process the top element of the EXEC stack, E:
 - (a) If E is an instruction: execute E (accessing whatever stacks are required).
 - (b) If E is a literal: push E onto the appropriate stack.
 - (c) If E is a list: push each element of E onto the EXEC stack, in reverse order.

(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)



exec code bool int float

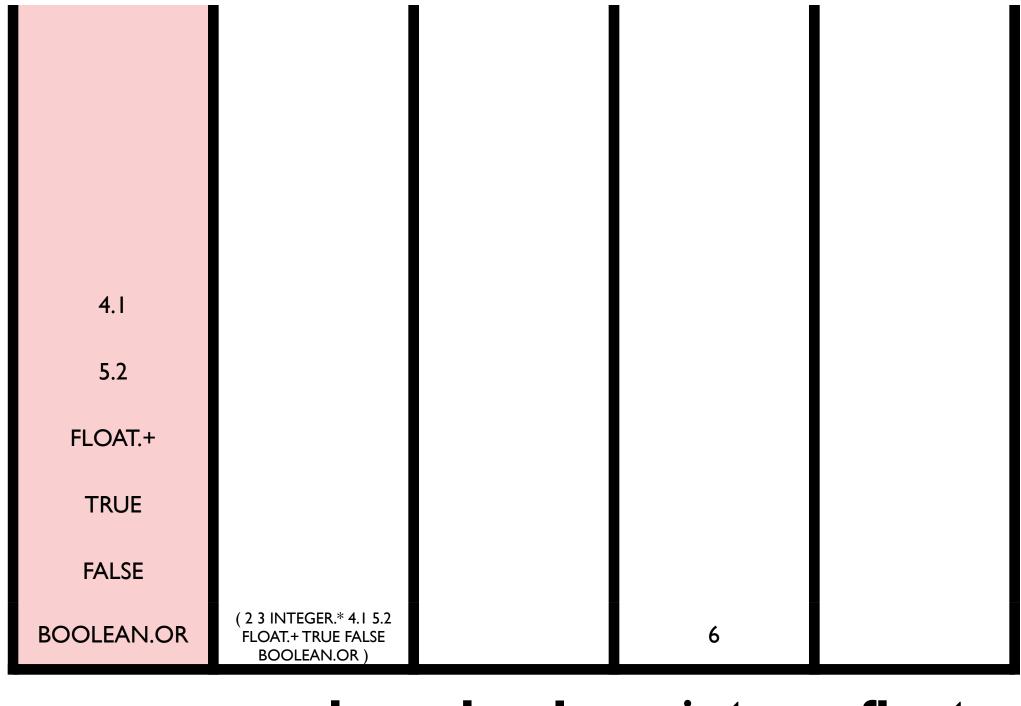
2			
3			
INTEGER.*			
4.1			
5.2			
FLOAT.+			
TRUE			
FALSE			
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)		

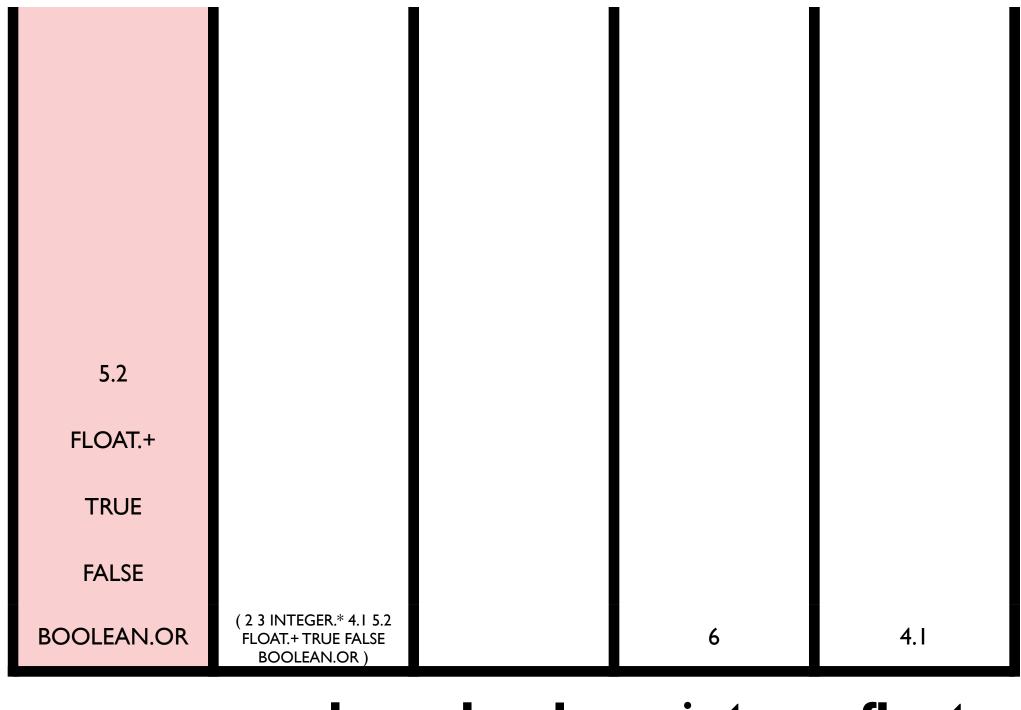
exec code bool int float

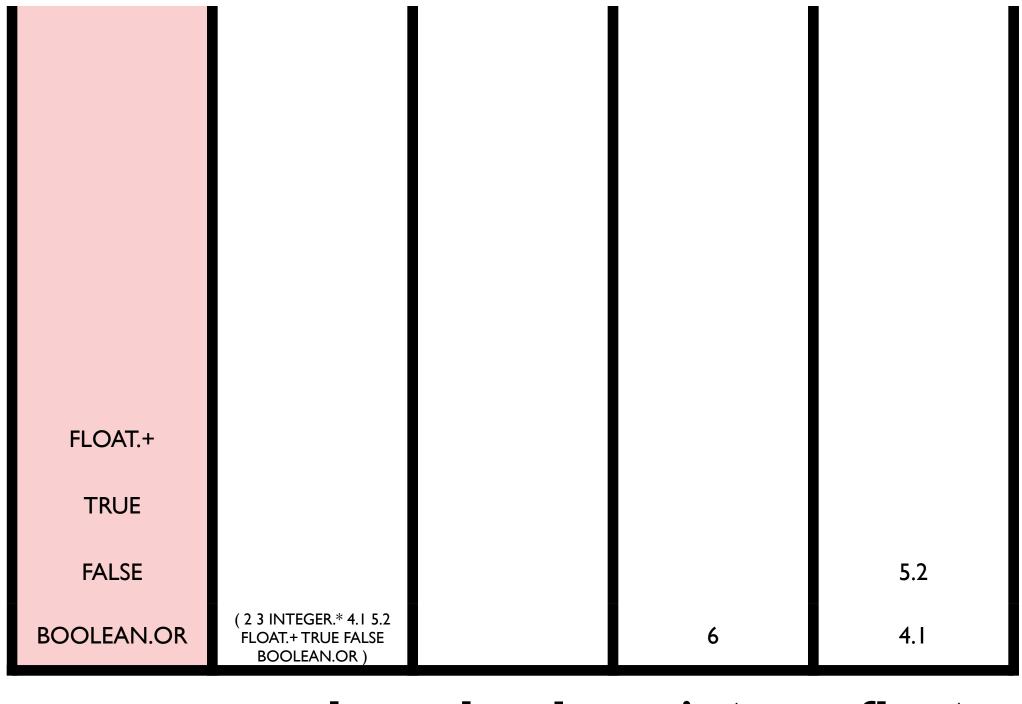
	BOOLEAN.OR)	<u> </u>	£laak
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE	2	
FALSE			
TRUE			
FLOAT.+			
5.2			
4.1			
INTEGER.*			
3			

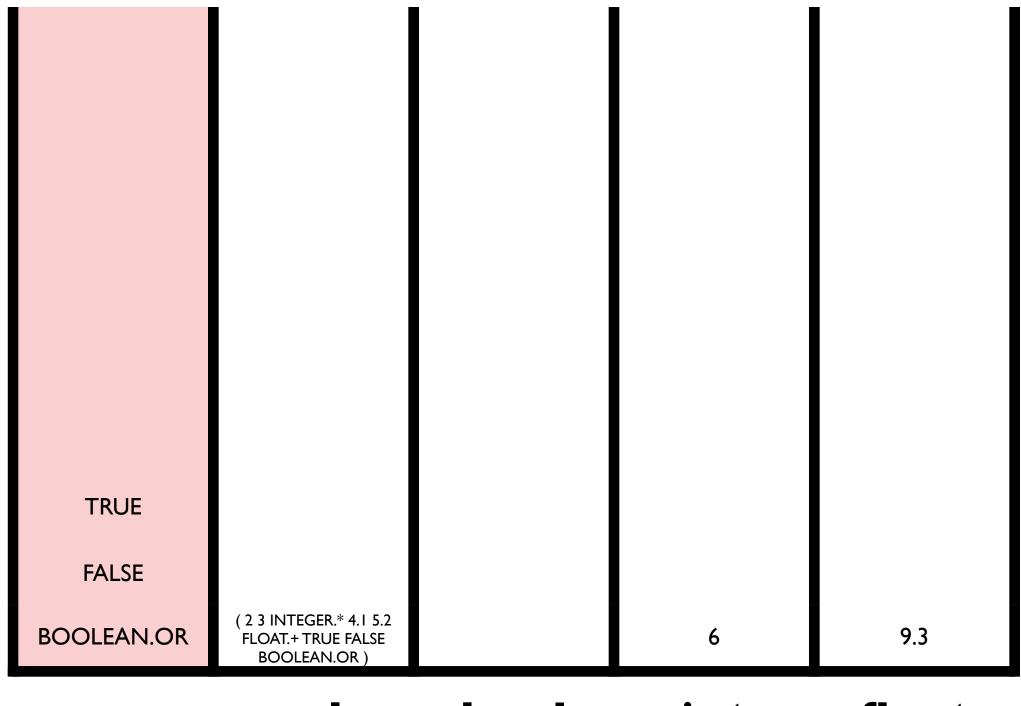
exec code bool int float

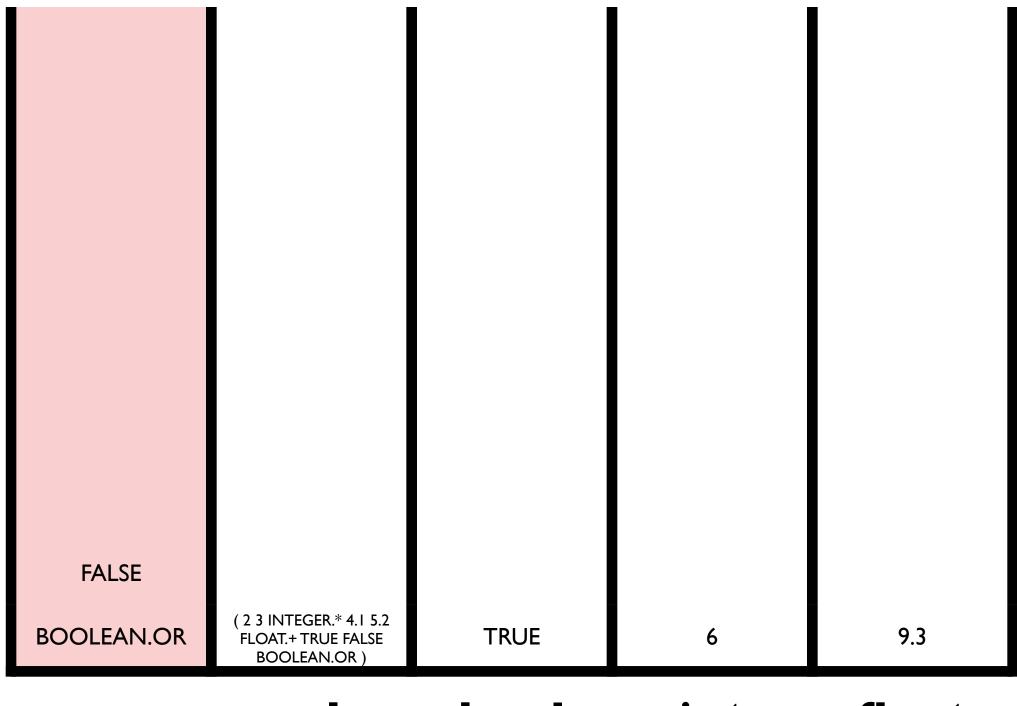
INTEGER.*				
4.1				
5.2				
FLOAT.+				
TRUE				
FALSE			3	
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)		2	
		_		<u> </u>



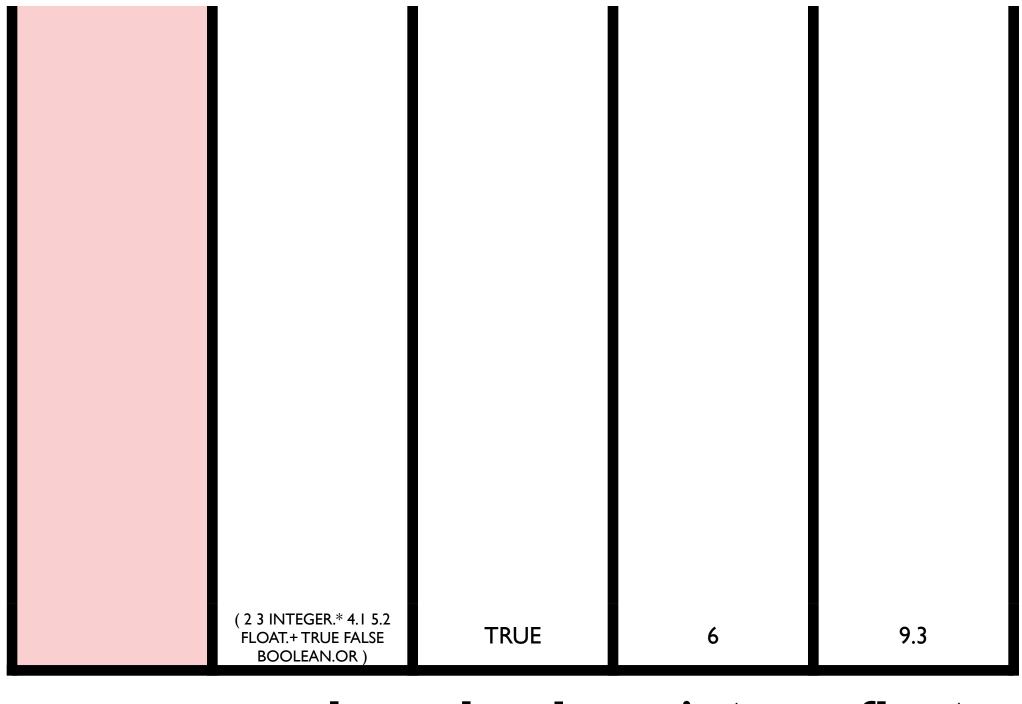








BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE	FALSE TRUE	6	9.3



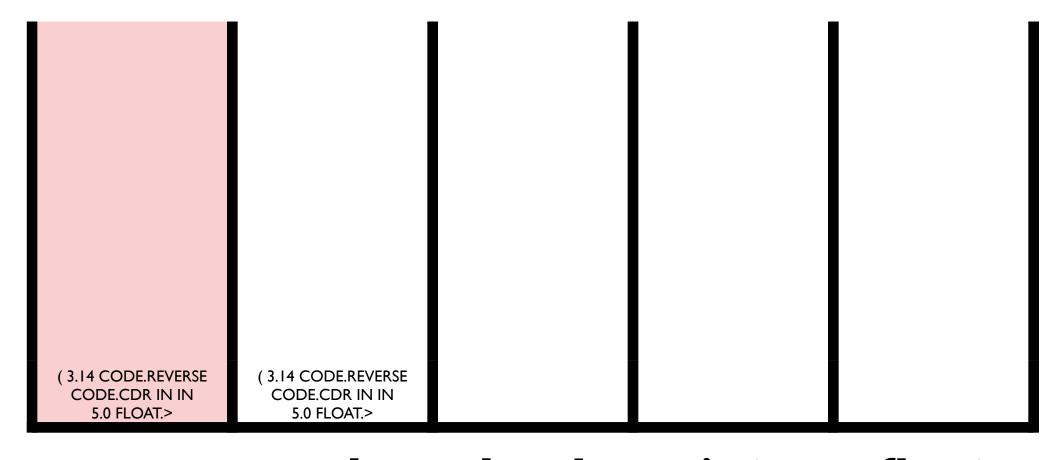
Same Results

```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+
TRUE FALSE BOOLEAN.OR )
```

```
( 2 BOOLEAN.AND 4.1 TRUE INTEGER./ FALSE 3 5.2 BOOLEAN.OR INTEGER.* FLOAT.+ )
```

(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.> (CODE.QUOTE FLOAT.*) CODE.IF)

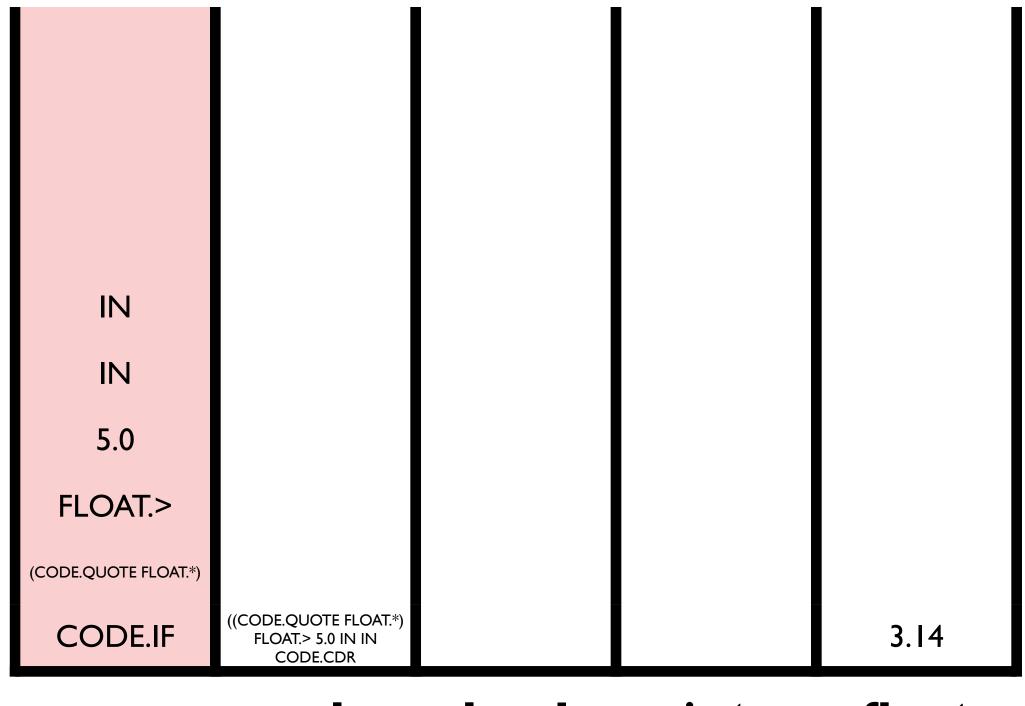
IN=4.0

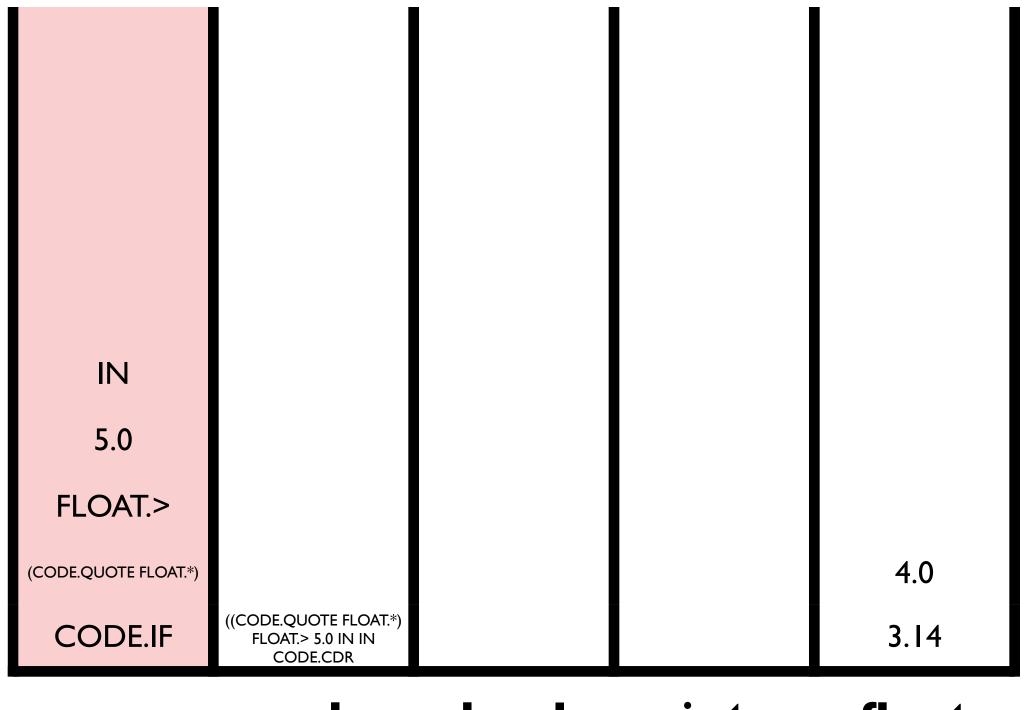


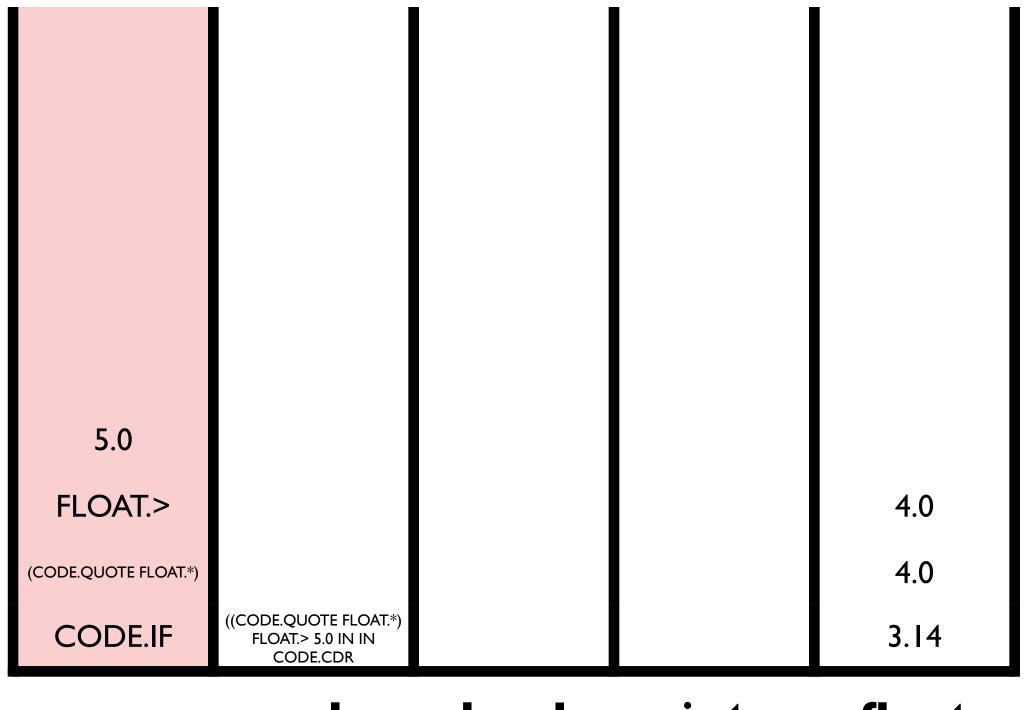
OVOC	codo	hool	int	float
CODE.IF	(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.>			
(CODE.QUOTE FLOAT.*)				
FLOAT.>				
5.0				
IN				
IN				
CODE.CDR				
CODE.REVERSE				
3.14				

CODE.IF	(3.14 CODE.REVERSE CODE.CDR IN IN 5.0 FLOAT.>		3.14
(CODE.QUOTE FLOAT.*)			
FLOAT.>			
5.0			
IN			
IN			
CODE.CDR			
CODE.REVERSE			

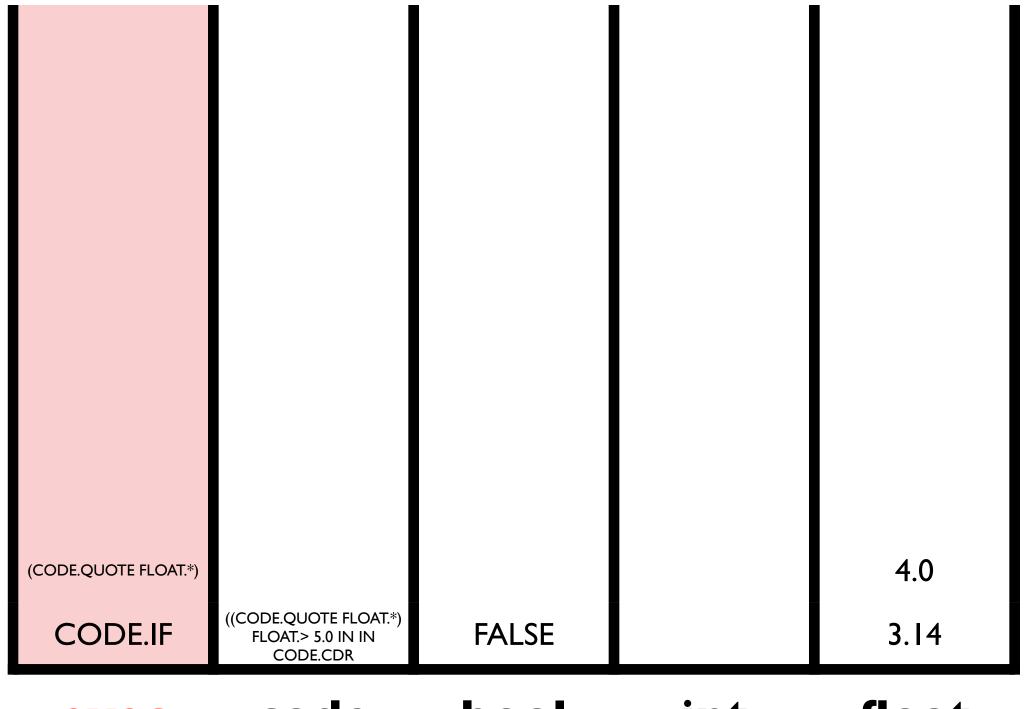
	CODE.IF	(CODE.IF (CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR		3.14
	(CODE.QUOTE FLOAT.*)			
	FLOAT.>			
	5.0			
	IN			
ı	IN			
ı	CODE.CDR			

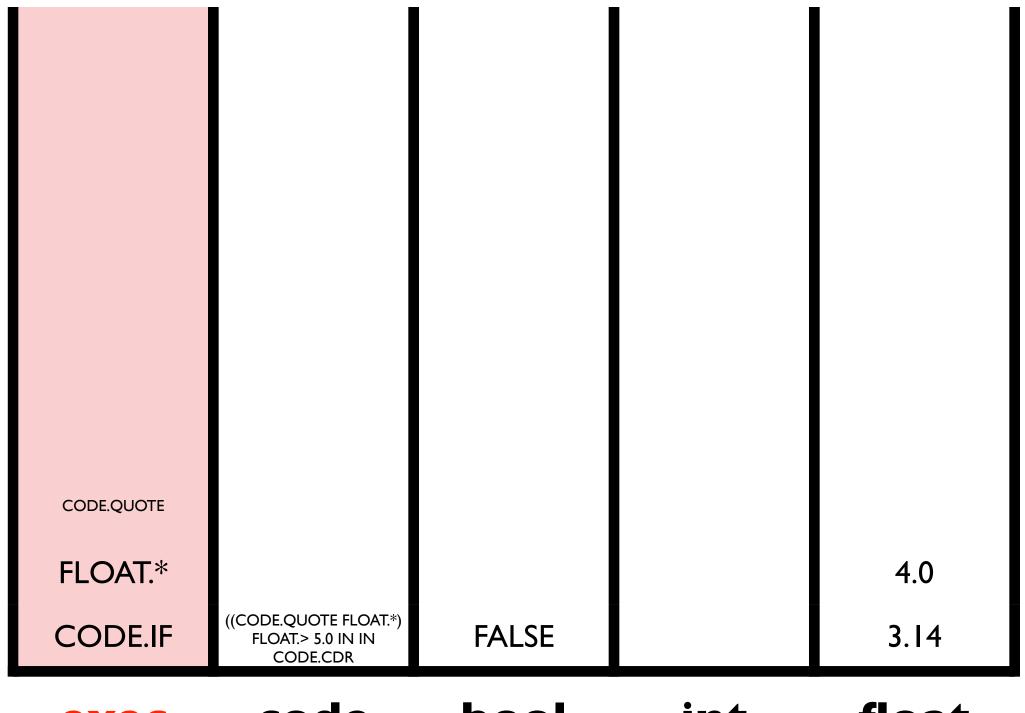


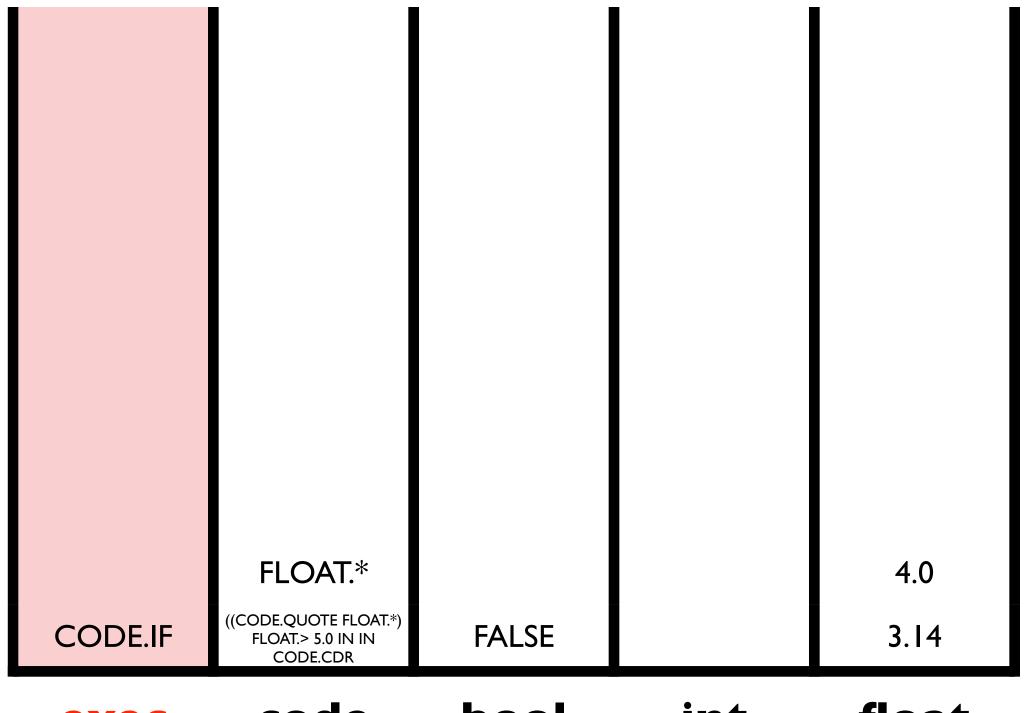


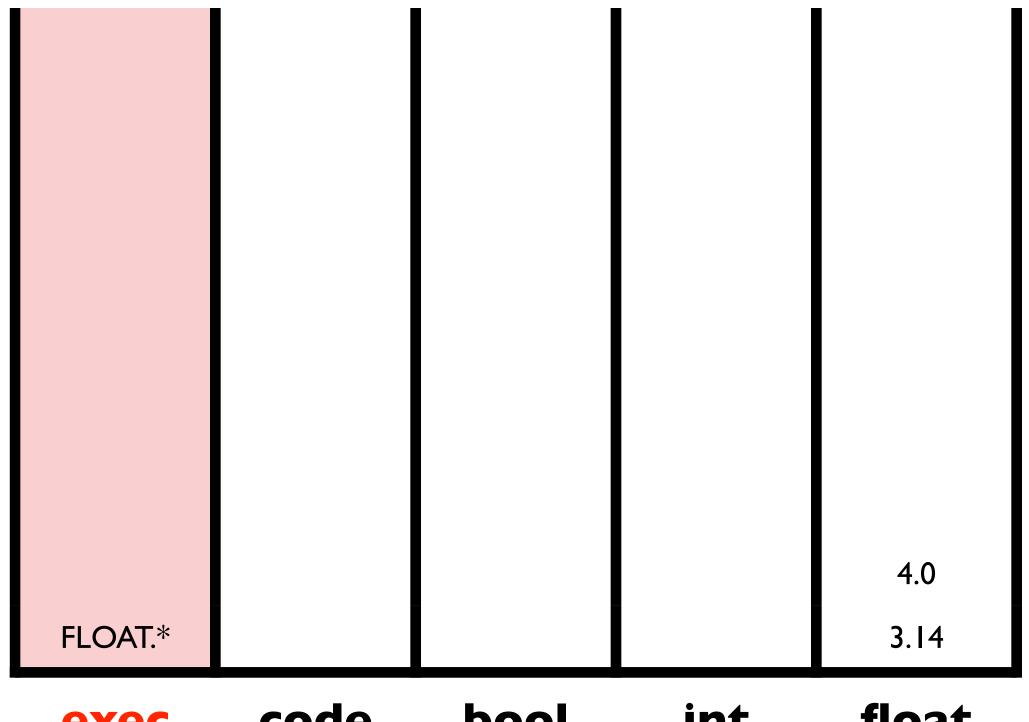


CODE.IF	((CODE.QUOTE FLOAT.*) FLOAT.> 5.0 IN IN CODE.CDR		3.14
(CODE.QUOTE FLOAT.*)			4.0
FLOAT.>			4.0
			5.0







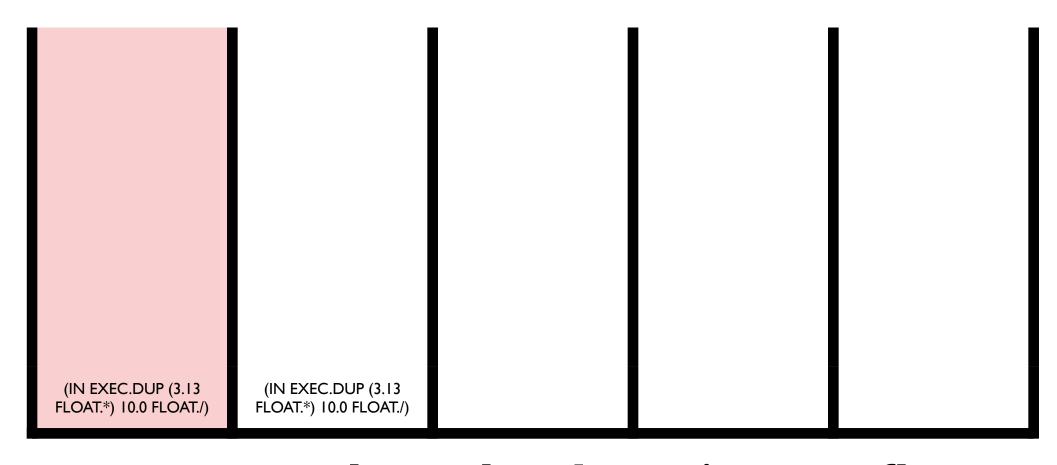


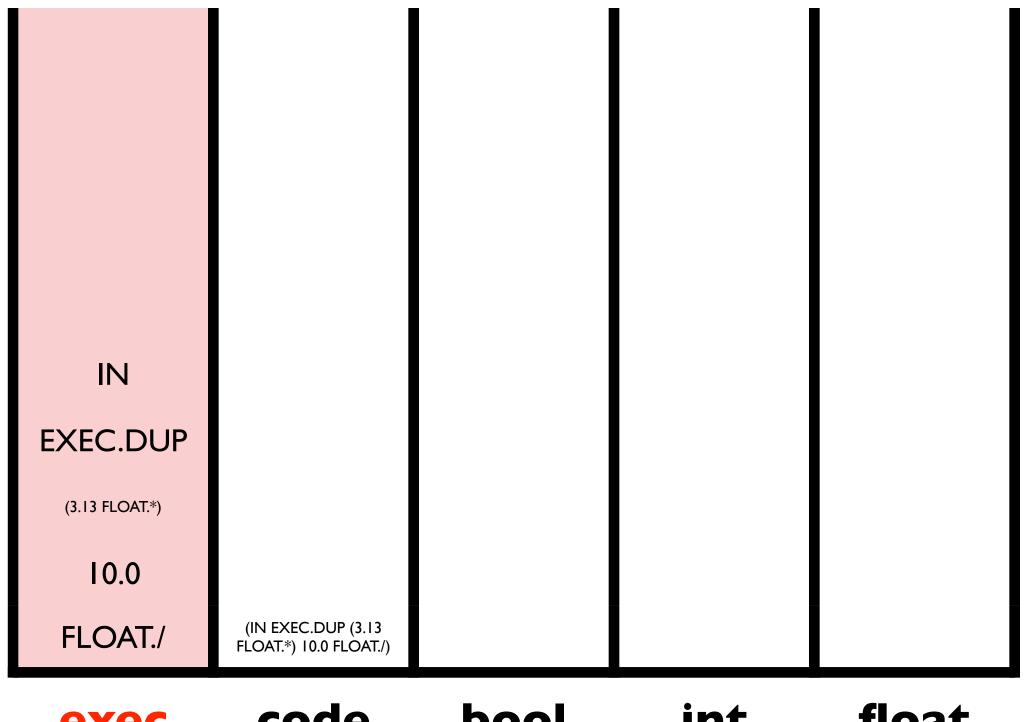
bool int code float exec



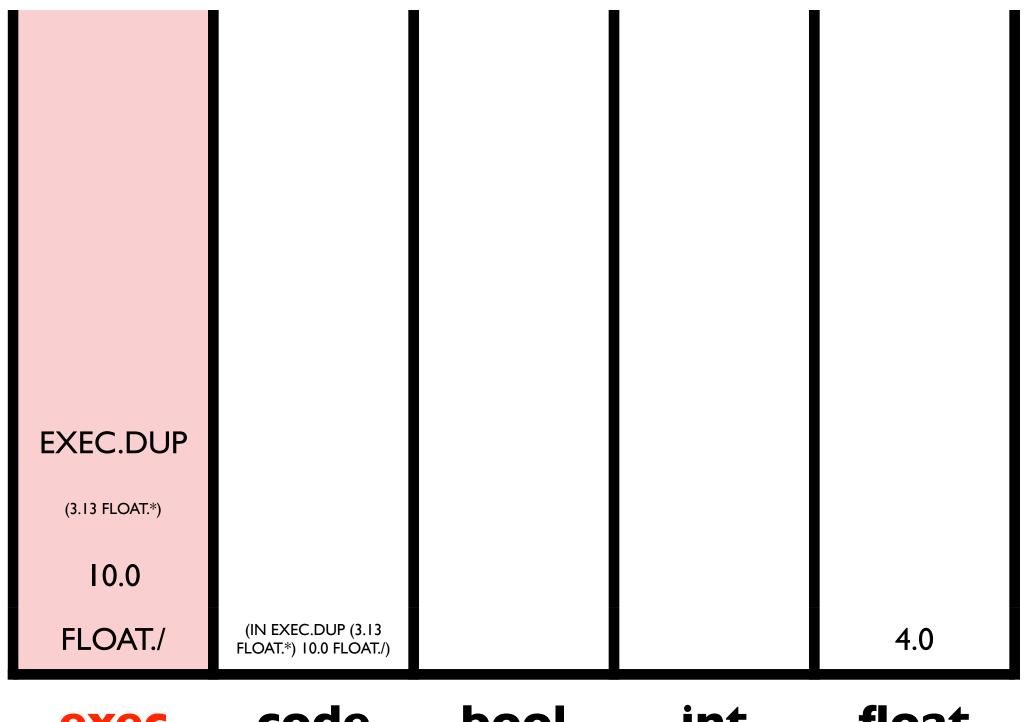
bool int code float exec

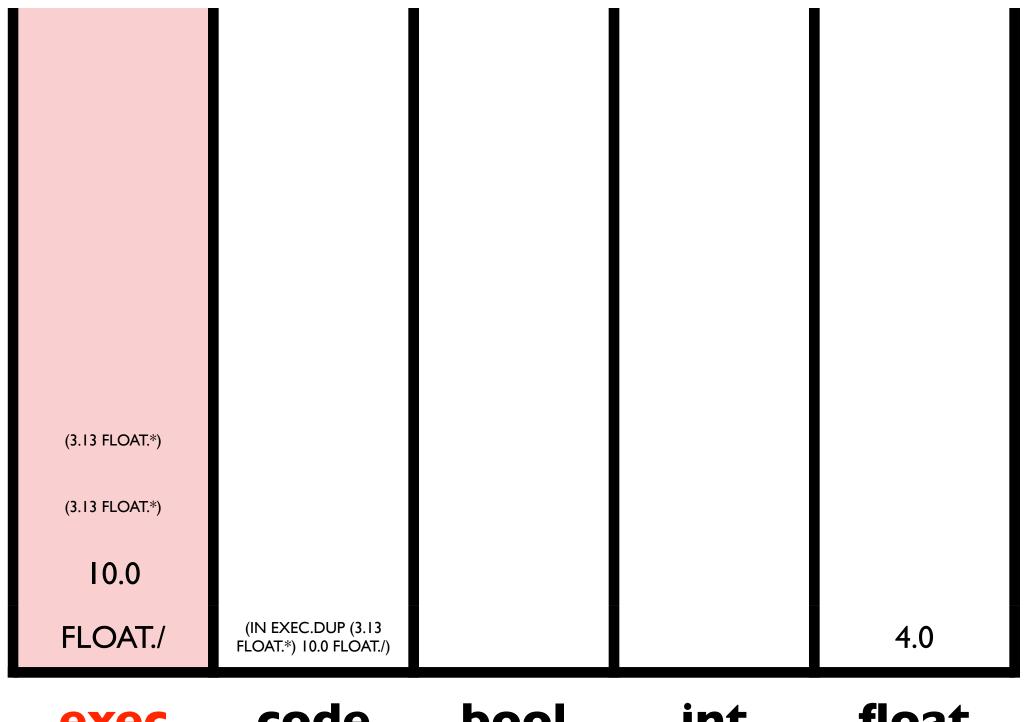
IN=4.0



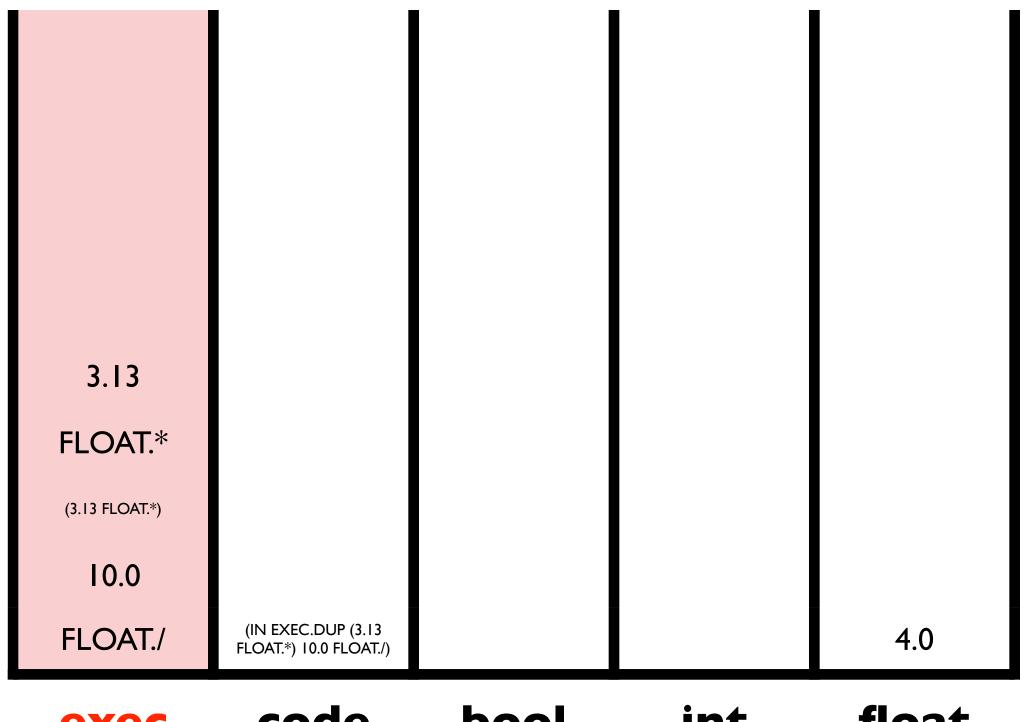


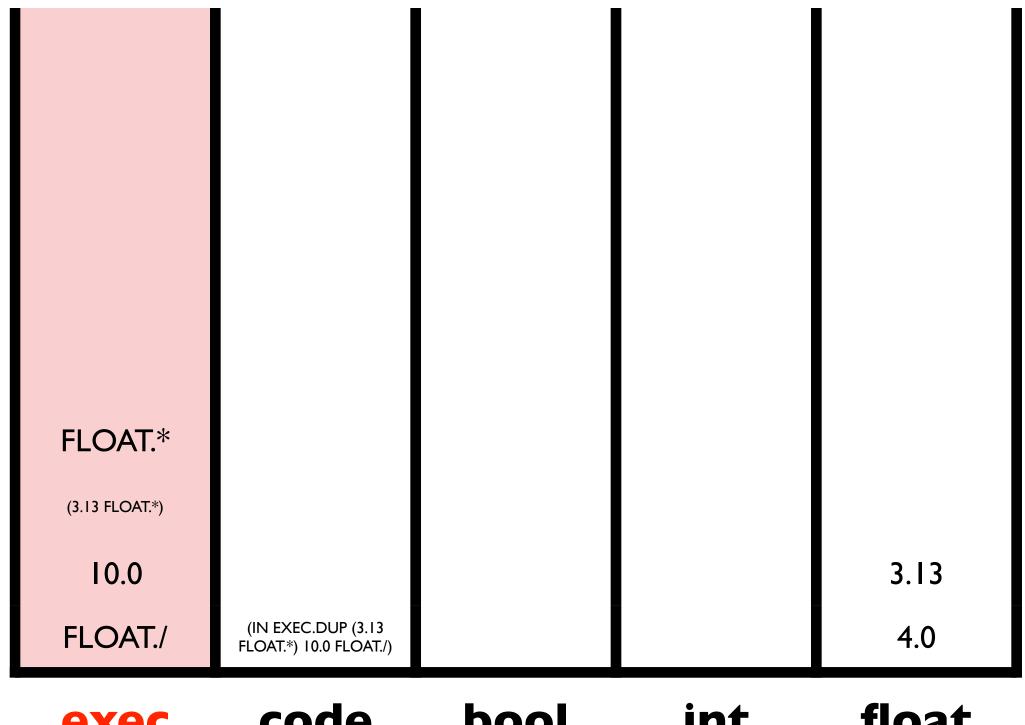
int float bool code exec



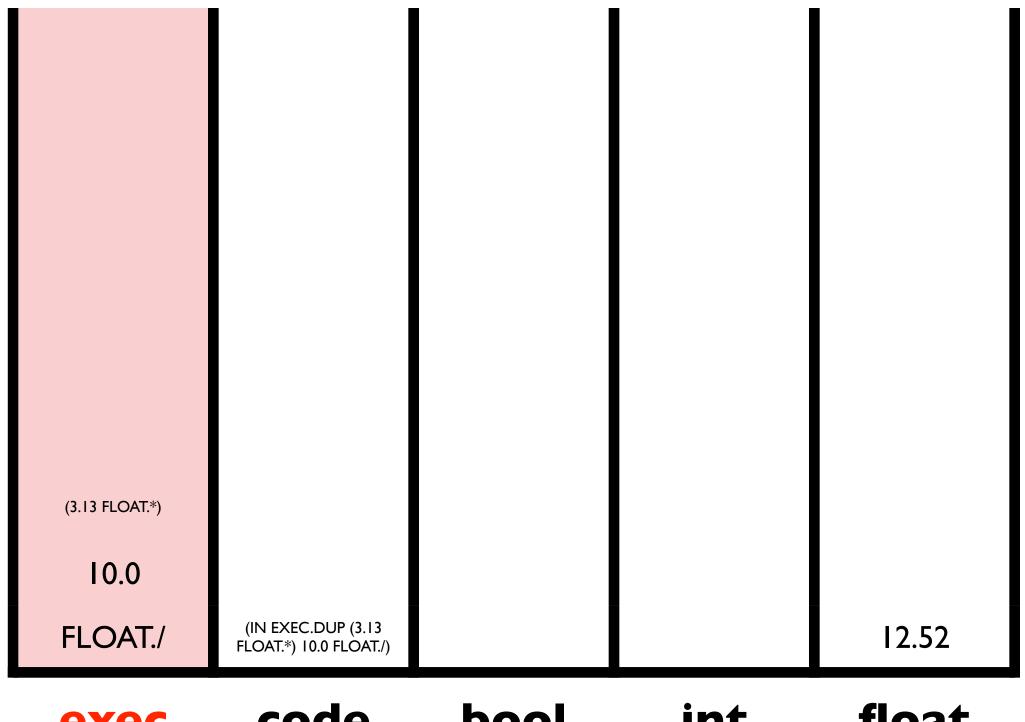


float int code bool exec

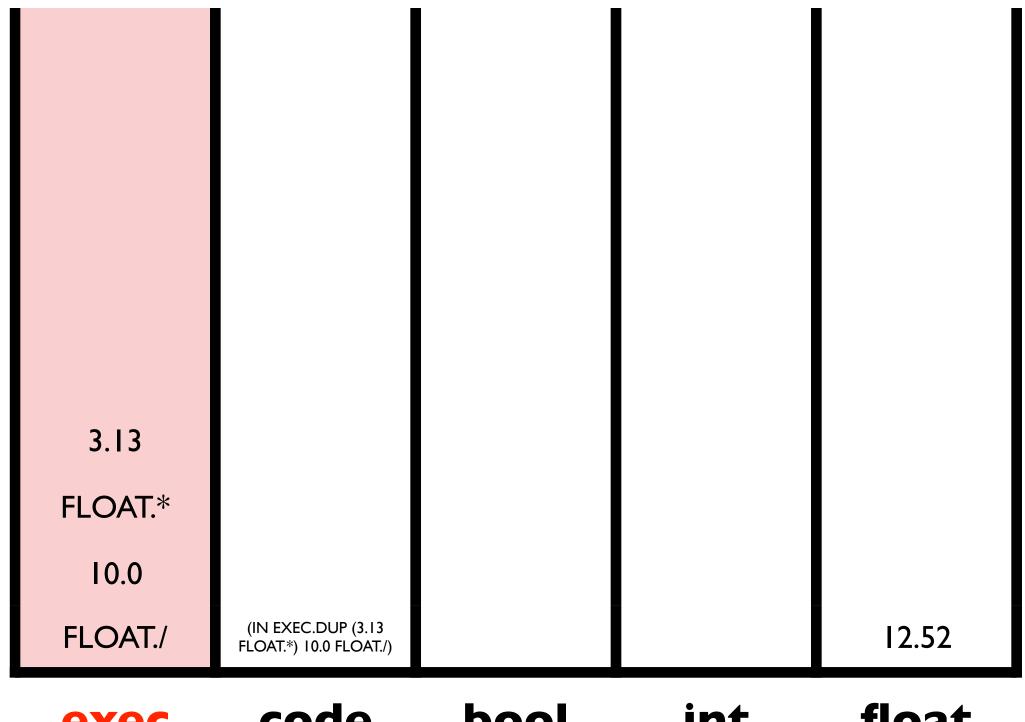




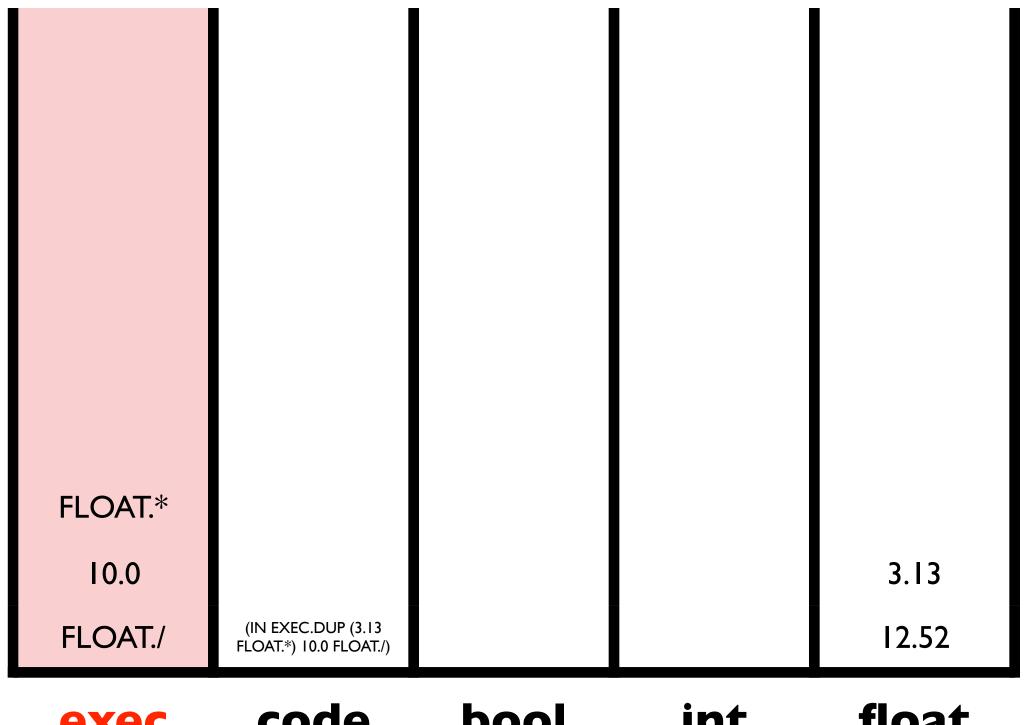
int bool float code exec



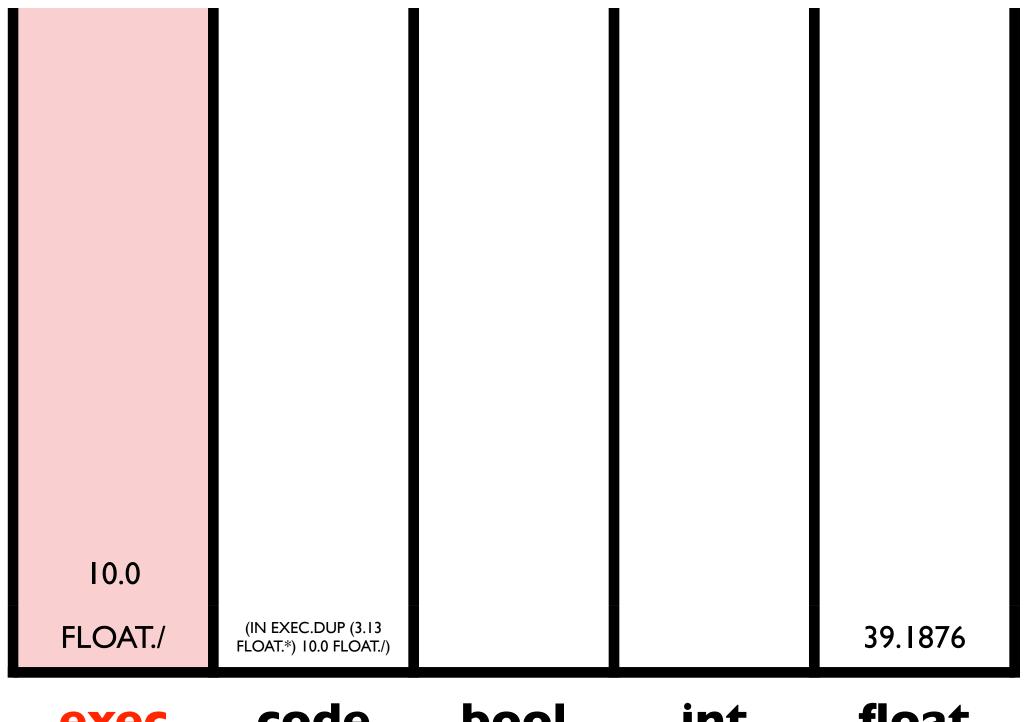
float int code bool exec



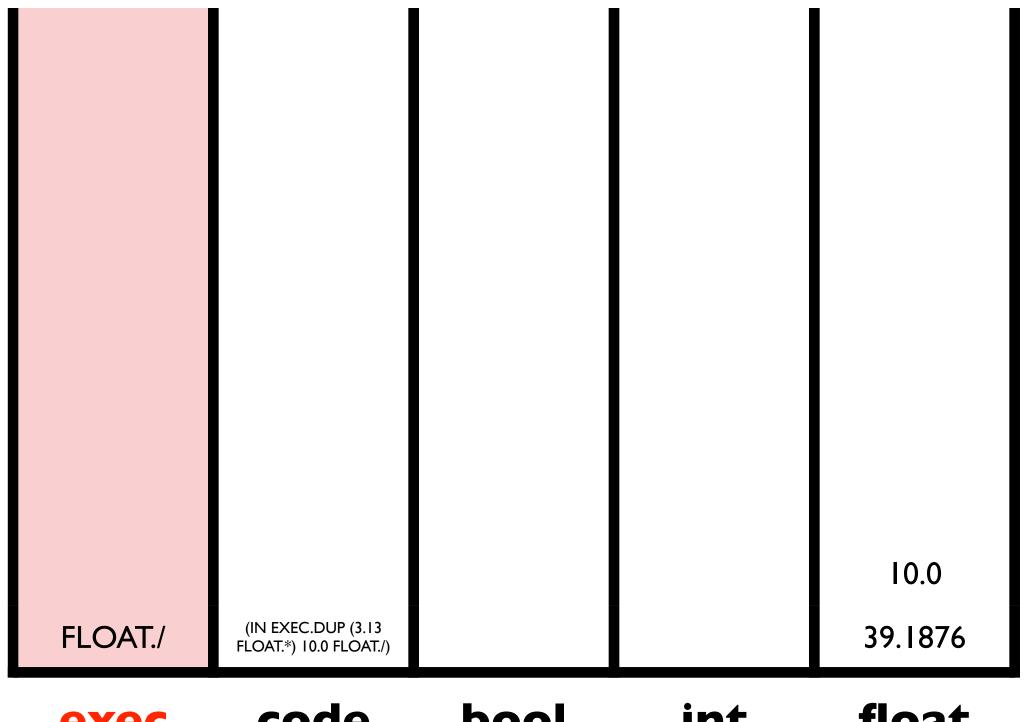
float int bool code exec



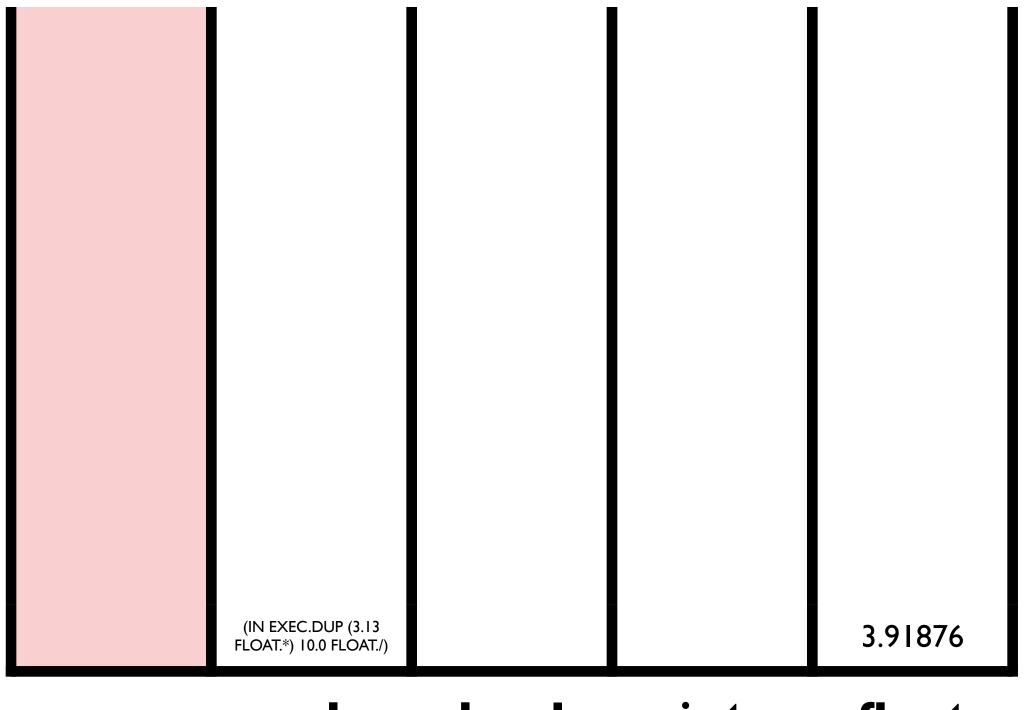
int float bool code exec



float int code bool exec



int code bool float exec



The Odd Problem

- Integer input
- Boolean output
- Was the input odd?
- ((code.nth) code.atom)

Combinators

- Standard K, S, and Y combinators:
 - EXEC.K removes the second item from the EXEC stack.
 - EXEC.S pops three items (call them A, B, and C) and then pushes (B C), C, and then A.
 - ullet EXEC.Y inserts (EXEC.Y T) under the top item (T).
- A *Y*-based "while" loop:

```
( EXEC.Y
  ( <BODY/CONDITION> EXEC.IF
  ( ) EXEC.POP ) )
```

Iterators

CODE.DO*TIMES, CODE.DO*COUNT, CODE.DO*RANGE

EXEC.DO*TIMES, EXEC.DO*COUNT, EXEC.DO*RANGE

Additional forms of iteration are supported through code manipulation (e.g. via CODE.DUP CODE.APPEND CODE.DO)

With Code Manipulation

- Transform code as data on "code" stack
- Execute transformed code with code.do, etc.
- Simple uses of modules can be evolved easily

With Execution Stack Manipulation

- Code queued for execution is stored on an "execution stack"
- Allow programs to duplicate and manipulate code that on the stack
- Example: (3 exec.dup (1 integer.+))

With Named Modules

- Uses Push's "name" stack
- Example:

```
(plus1 exec.define (1 integer.+))
...
plus1
```

Coordinating definitions/references is tricky

Holland's Tags

- Initially arbitrary identifiers that come to have meaning over time
- Matches may be inexact
- Appear to be present in some form in many different kinds of complex adaptive systems
- Examples range from immune systems to armies on a battlefield
- A general tool for the support of emergent complexity

With tags

- Include instructions that tag code (modules)
- Include instructions that recall and execute modules by *closest matching* tag
- If a single module has been tagged then all tag references will recall modules
- The number of tagged modules can grow incrementally over evolutionary time
- Expressive and evolvable

Tags in Push

- Tags are integers embedded in instruction names
- Instructions like tag.exec.123 tag values
- Instructions like tagged.456 recall values by closest matching tag
- If a single value has been tagged then all tag references will recall (and execute) values
- The number of tagged values can grow incrementally over evolutionary time

Tags in Trees

• Example:

- Must do something about endless recursion
- Must do something about return values of tagging operations and references prior to tagging
- Non-trivial to support arguments in a general way
- Utility not clear from experiments conducted to date

Auto-simplification

Loop: Make it randomly simpler

If it's as good or better: keep it

Otherwise: revert

GECCO-2014 poster showed that this can efficiently and reliably reduce the size of the evolved programs

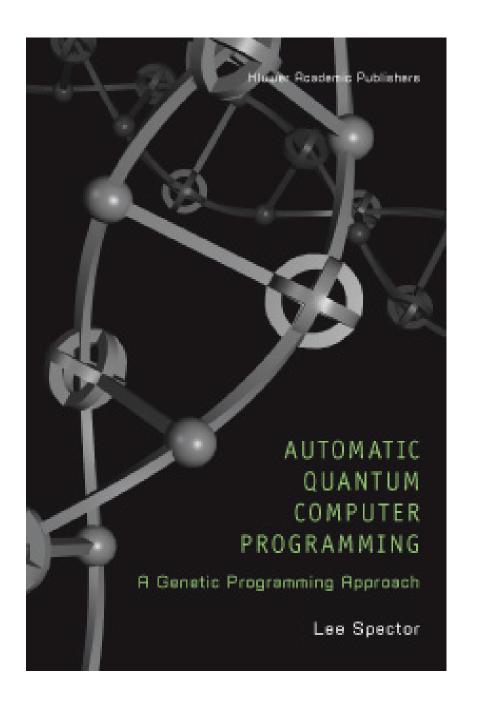
GECCO-2014 student paper explored its utility in a genetic operator

Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting

29 Synthesis Benchmarks

- Number IO, Small or Large, For Loop Index, Compare
 String Lengths, Double Letters, Collatz Numbers, Replace
 Space with Newline, String Differences, Even Squares,
 Wallis Pi, String Lengths Backwards, Last Index of Zero,
 Vector Average, Count Odds, Mirror Image, Super
 Anagrams, Sum of Squares, Vectors Summed, X-Word
 Lines, Pig Latin, Negative to Zero, Scrabble Score, Word
 Stats, Checksum, Digits, Grade, Median, Smallest, Syllables
- PushGP has solved all of these except for the ones in blue
- Presented in a GECCO-2015 GP track paper



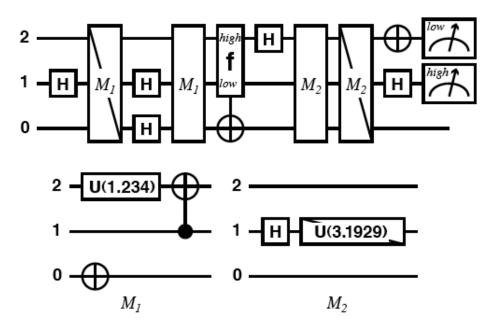


Figure 8.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with M_1 and M_2 standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate symbol indicates that the matrix for the gate is transposed. The "f" gate is the oracle.

Humies 2004 GOLD MEDAL

Genetic Programming for Finite Algebras

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Autoconstructive Evolution

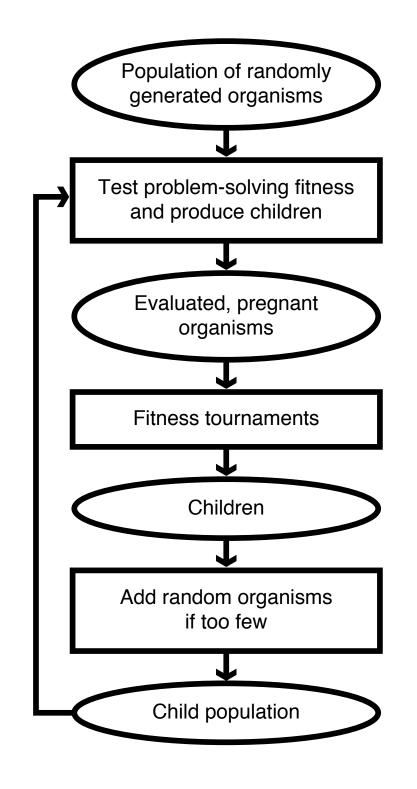
- Individuals make their own children
- Agents thereby control their own mutation rates, sexuality, and reproductive timing
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- Radical self-adaptation

Related Work

- MetaGP: but (1) programs and reproductive strategies dissociated and (2) generally restricted reproductive strategies
- ALife systems such as Tierra, Avida, SeMar: but
 (1) hand-crafted ancestors, (2) reliance on cosmic ray mutation, and (3) weak problem solving
- Evolved self-reproduction: but generally exact reproduction, non-improving (exception: Koza, but very limited tools for problem solving *and* for construction of offspring)

Pushpop

- A soup of evolving Push programs
- Reproductive procedures emerge ex nihilo:
 - No hand-designed "ancestor"
 - Children constructed by any computable process
 - No externally applied mutation procedure or rate
 - Exact clones are prohibited, but near-clones are permitted.
- Selection for problem-solving performance



Pushpop Results

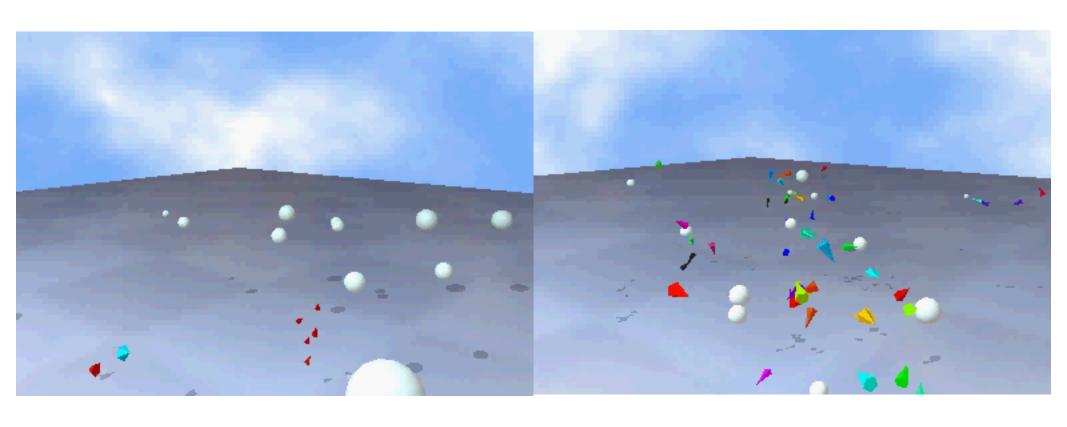
- In adaptive populations:
 - Species are more numerous
 - Diversification processes are more reliable
- Selection can promote diversity
- Provides a possible explanation for the evolution of diversifying reproductive systems

SwarmEvolve 2.0

- Behavior (including reproduction) controlled by evolved Push programs
- Color, color-based agent discrimination controlled by agents
- Energy conservation
- Facilities for communication, energy sharing
- Ample user feedback (e.g. diversity metrics, agent energy determines size)

Instruction(s)	Description
DUP, POP, SWAP, REP, =, NOOP, PULL, PULLDUP, CONVERT, CAR, CDR, QUOTE, ATOM, NULL, NTH, +, *, /, >, <, NOT, AND, NAND OR, NOR, DO*, IF	Standard Push instructions (See [11])
VectorX, VectorY, VectorZ, VPlus, VMinus, VTimes, VDivide,	Vector access, construction, and manipulation
VectorLength, Make-Vector	-
RandI, RandF, RandV, RandC	Random number, vector, and code generators
SetServoSetpoint, SetServoGain,	Servo-based persistent
Servo	memory
Mutate, Crossover	Stochastic list manipulation
Spawn	(parameters from stacks) Produce a child with code from code stack
ToFood	Vector to energy source
FoodIntensity	Energy of energy source
MyAge, MyEnergy, MyHue, MyVelocity, MyLocation, MyProgram	Information about self
ToFriend, FriendAge, FriendEnergy,	Information about closest
FriendHue, FriendVelocity,	agent of similar hue
FriendLocation, FriendProgram	
ToOther, OtherAge, OtherEnergy, OtherHue, OtherVelocity,	Information about closest
OtherLocation, OtherProgram	agent of non-similar hue
FeedFriend, FeedOther	Transfer energy to closest agent of indicated category

SwarmEvolve 2.0



Winner, Best Paper Award, AAAA Track, GECCO-2003

Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time

Future Work

- Expression of variable scope and environments (implemented in Push, but not yet studied systematically)
- Expression of concurrency and parallelism
- Applications for which expressiveness is likely to be essential, e.g. complete software applications, agents in complex, dynamic environments
- Epigenetics

Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be unusually simple and powerful
- Push is expressive, evolvable, successful, and extensible
- http://pushlanguage.org

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