



# Expressive Genetic Programming: Concepts and Applications

A Tutorial at the Genetic and Evolutionary Computation Conference (GECCO-2017, Berlin)

*This is an updated version of the tutorial of the same name from GECCO-2016*

Lee Spector

School of Cognitive Science  
Hampshire College  
Amherst, MA USA  
lspector@hampshire.edu

Nicholas Freitag McPhee

Division of Science & Mathematics  
University of Minnesota, Morris  
Morris, Minnesota USA  
mcphee@morris.umn.edu

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# Instructors (1)



Lee Spector is a Professor of Computer Science in the School of Cognitive Science at Hampshire College in Amherst, Massachusetts, and an adjunct professor in the Department of Computer Science at the University of Massachusetts, Amherst. He received a B.A. in Philosophy from Oberlin College in 1984 and a Ph.D. from the Department of Computer Science at the University of Maryland in 1992. His areas of teaching and research include genetic and evolutionary computation, quantum computation, and a variety of intersections between computer science, cognitive science, evolutionary biology, and the arts. He is the Editor-in-Chief of the journal *Genetic Programming and Evolvable Machines* (published by Springer) and a member of the editorial board of *Evolutionary Computation* (published by MIT Press). He is also a member of the SIGEVO executive committee and he was named a Fellow of the International Society for Genetic and Evolutionary Computation.

More info: <http://hampshire.edu/l spectator>

## Instructors (2)



Nicholas Freitag McPhee is a Professor of Computer Science in the Division of Science and Mathematics at the University of Minnesota, Morris, in Morris, Minnesota. He received a B.A. in Mathematics from Reed College in 1986 and a Ph.D. from the Department of Computer Sciences at the University of Texas at Austin in 1993. His areas of teaching and research include genetic and evolutionary computation, machine learning, software development, and, when circumstances allow, photography and American roots music. He is a co-author of *A field guide to genetic programming*, and a member of the editorial board of the journal *Genetic Programming and Evolvable Machines* (published by Springer).

More info: <http://facultypages.morris.umn.edu/~mcphee>

# Scope and Content (1)

The language used to express evolving programs can significantly impact the dynamics and problem-solving capabilities of a genetic programming system. In GP these impacts are driven by far more than the absolute computational power of the languages used; just because a computation is theoretically possible in a language, doesn't mean it's readily discoverable or leveraged by evolution. Highly expressive languages can facilitate the evolution of programs for any computable function using, as appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

This tutorial will present a range of approaches that have been taken for evolving programs in expressive programming languages. We will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, supporting the evolution of complex, modular programs in a particularly simple and flexible way.

# Scope and Content (2)

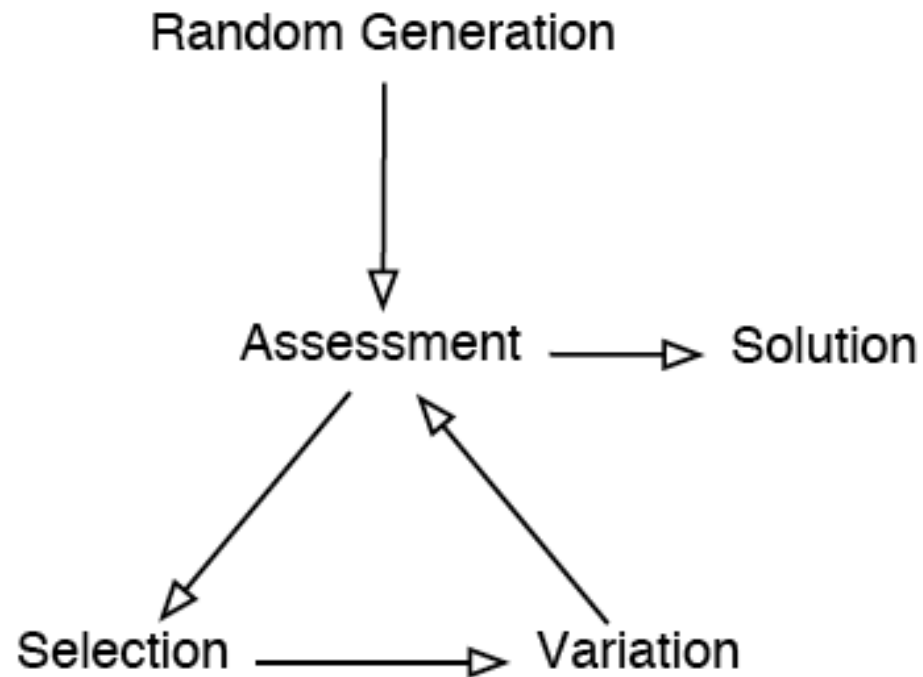
Interleaved with our description of the Push language will be demonstrations of the use of analytical tools such as graph databases to explore ways in which evolved Push programs are actually taking advantage of the language's expressive features. We will illustrate, for example, the effective use of multiple types and type-appropriate functions, the evolution and modification of code blocks and looping/recursive constructs, and the ability of Push programs to handle multiple, potentially related tasks.

We'll conclude with a brief review of over a decade of Push-based research, including the production of human-competitive results, along with recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

# Course Agenda

- Genetic programming
- Expressiveness and evolvability
- The Push programming language and PushGP
- Demos
- Results
- Ongoing Research

# Evolutionary Computation



# Genetic Programming

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software



# Program Representations

- Lisp-style symbolic expressions (Koza, ...)
- Purely functional/lambda expressions (Walsh, Yu, ...)
- Linear sequences of machine/byte code (Nordin et al., ...)
- Artificial assembly-like languages (Ray, Adami, ...)
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...)
- Graph-structured programs (Teller, Globus, ...)
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...)
- Fuzzy rule systems (Tunstel, Jamshidi, ...)
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...)
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, McKay, ...)

# Mutating Lisp (1)

```
(+ (* X Y)  
  (+ 4 (- Z 23)))
```

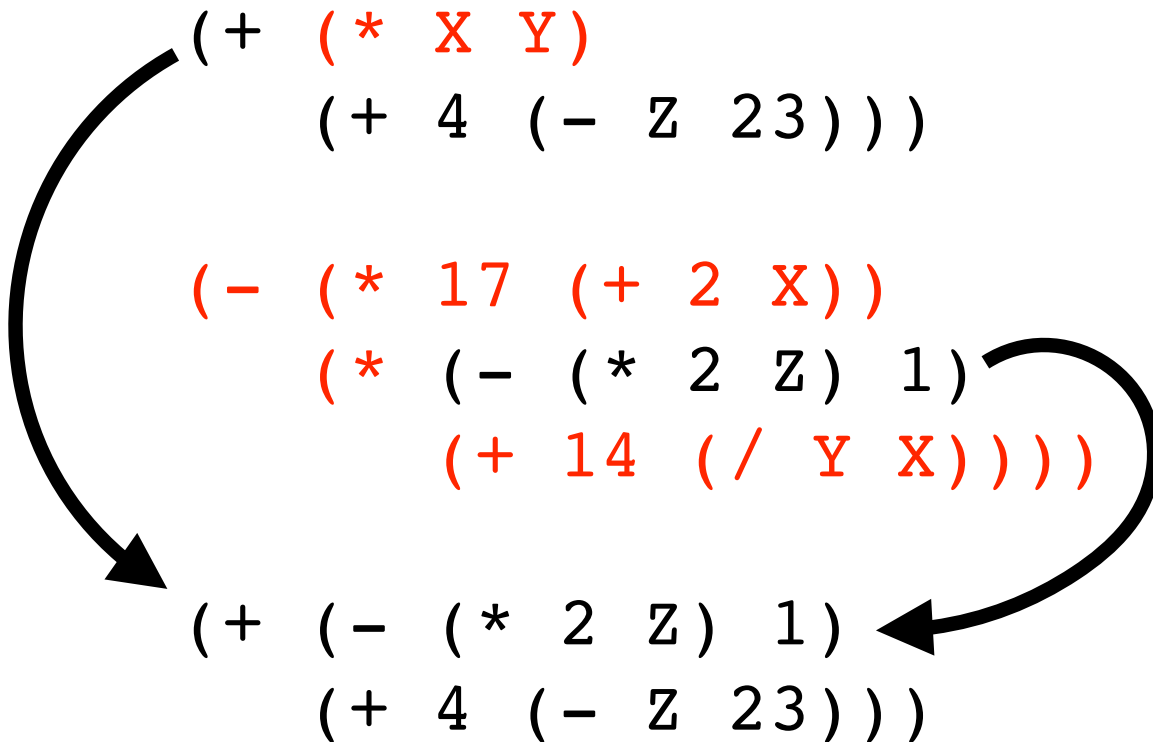
# Mutating Lisp (2)

```
(+ (* X Y)  
  (+ 4 (- Z 23)))
```

# Mutating Lisp (3)

```
(+ (- (+ 2 2) z)  
   (+ 4 (- z 23)))
```

# Recombining Lisp



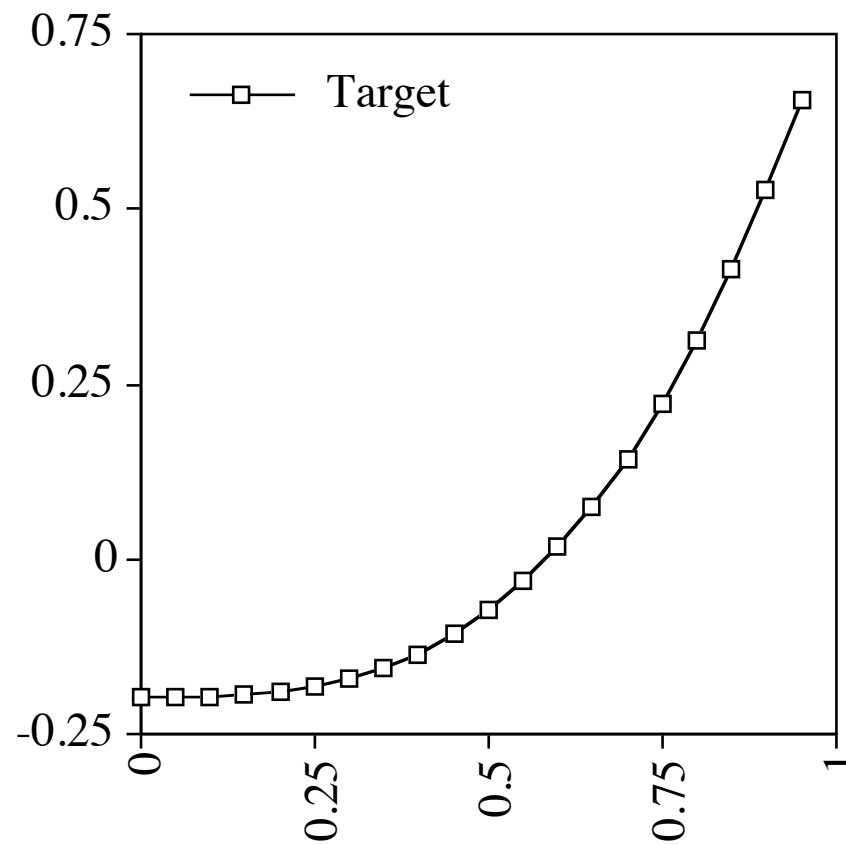
# Symbolic Regression

- A simple example
- Given a set of data points, evolve a program that produces  $y$  from  $x$ .
- Primordial ooze:  $+$ ,  $-$ ,  $*$ ,  $\%$ ,  $x$ ,  $0.1$
- Fitness = error (smaller is better)

# Genetic Programming Parameters

- Maximum number of Generations: 51
- Size of Population: 1000
- Maximum depth of new individuals: 6
- Maximum depth of new subtrees for mutants: 4
- Maximum depth of individuals after crossover: 17
- Fitness-proportionate reproduction fraction: 0.1
- Crossover at any point fraction: 0.3
- Crossover at function points fraction: 0.5
- Selection method: FITNESS-PROPORTIONATE
- Generation method: RAMPED-HALF-AND-HALF
- Randomizer seed: 1.2

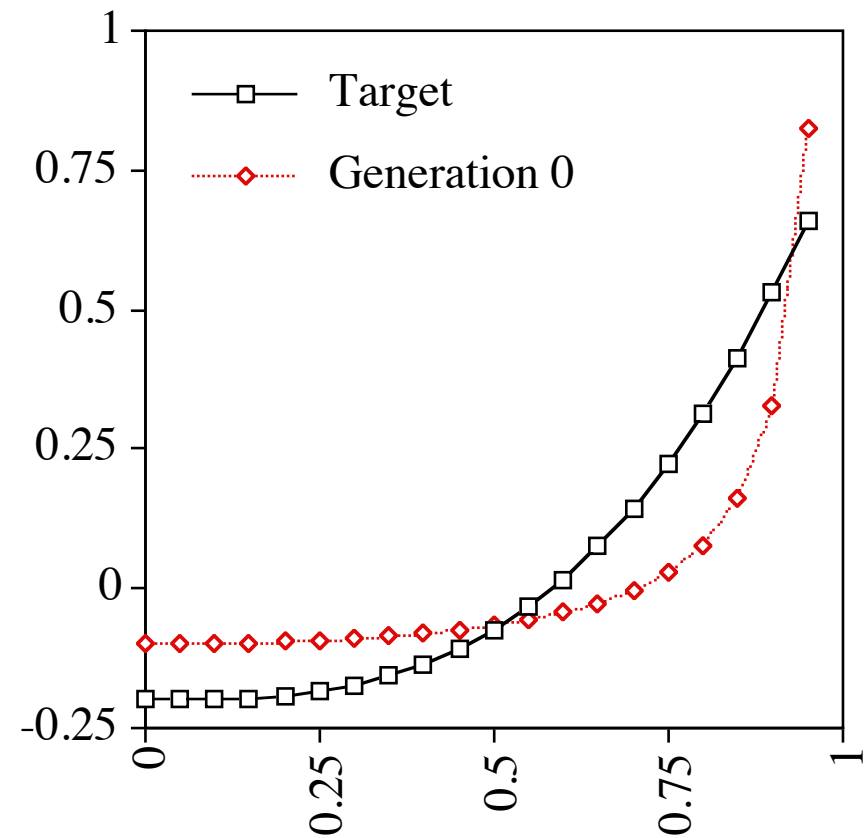
Evolving  $y = x^3 - 0.2$





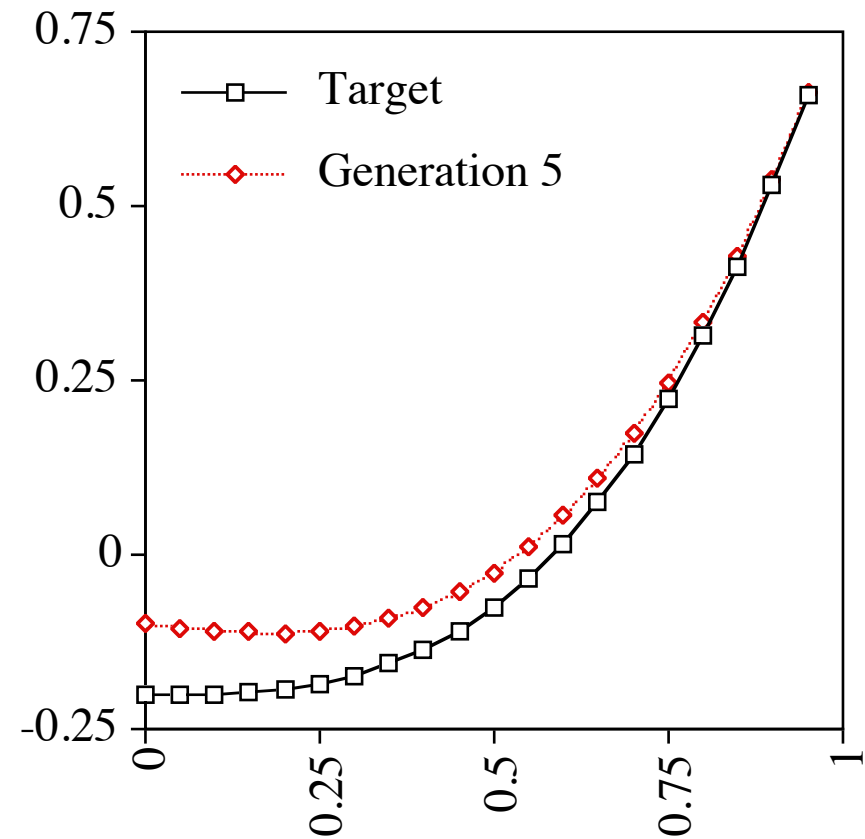
# Best Program, Gen 0

```
(- (% (* 0.1  
      (* X X) )  
  (- (% 0.1 0.1)  
      (* X X) ) )  
0.1)
```



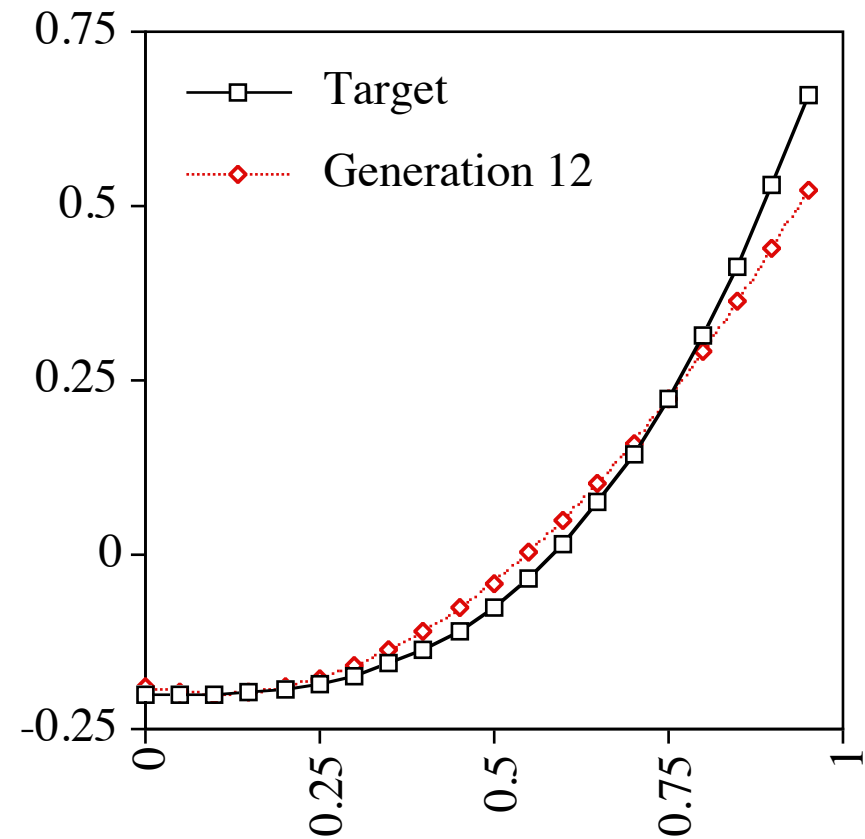
# Best Program, Gen 5

```
(- (* (* (% X 0.1)
          (* 0.1 X))
   (- X
      (% 0.1 X)))
0.1)
```



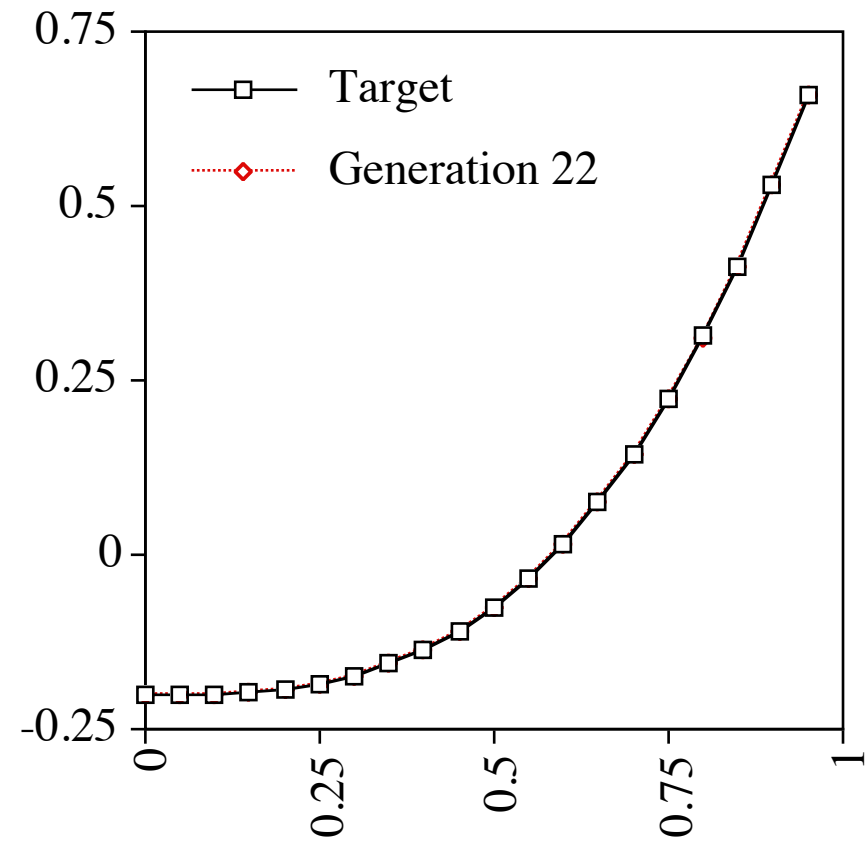
# Best Program, Gen 12

```
(+ (- (- 0.1
      (- 0.1
        (- (* X X)
          (+ 0.1
            (- 0.1
              (* 0.1
                0.1)))))))
(* X
  (* (% 0.1
      (% (* (* (- 0.1 0.1)
              (+ X
                (- 0.1 0.1))))
        X)
      (+ X (+ (- X 0.1)
              (* X X))))))
(+ 0.1 (+ 0.1 X))))
(* X X))
```



# Best Program, Gen 22

```
(- (- (* X (* X X))  
      0.1)  
  0.1)
```



# Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Partial recursive functions
- Register machine programs
- Assembly language programs
- etc.

# Evolvability

- The fact that a computation **can be expressed** in a formalism does **not** imply that it **will be produced** in by a human programmer **or** by evolution.
- Research program:
  1. Provide expressiveness
  2. Study/enhance evolvability

# Data/Control Structure

- Data abstraction and organization

Data types, variables, data structures, name spaces, ...

- Control abstraction and organization

Conditionals, loops, modules, threads, ...

# Evolving Structure (1)

- Specialize GP techniques to **directly** support human programming language abstractions
- Examples:
  1. Structured data via strongly typed GP
  2. Structured control via automatically defined functions



# Strongly Typed GP (Montana)

- Primitives annotated with types
- Constrained code generation
- Constrained mutation and recombination

# Automatically Defined Functions (Koza)

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

## Evolving Structure (2)

- Specialize GP techniques to **indirectly** support human programming language abstractions
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

# Evolving Structure (3)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- Minimal syntax + maximal, flexible semantics
- **Push**

# Push (1)

- Designed for program evolution
- Data flows via stacks, not syntax
- One stack per type:  
integer, float, boolean, string, code, exec, vector, ...
- `program` → `instruction` | `literal` | `( program* )`
- Turing complete, with rich data and control structures

# Push (2)

- Missing argument? NOOP
- Argument order: Generally reflect expected order from text of canonical usage
- Time/step limits ensure termination, with results available from stacks in all cases
- PushGP is a GP system that evolves Push programs
- <http://pushlanguage.org>

# Push (3)

- It is simple to write a minimal Push system in any language; instructions can then be added incrementally
- Implementations in C++, C#, Clojure, Common Lisp, Java, Javascript, Python, Racket, Ruby, Scala, Scheme, Swift, ...
- Most examples in this presentation use Clojush, a Push/ PushGP implementation in Clojure
- A recent addition that might be more approachable is Pysh, a Push/ PushGP implementation in Python

 Inspector lein release :minor

b1c6802 on Jan 12

5 contributors



341 lines (276 sloc) | 17.9 KB

Raw

Blame

History



# Clojush

build passing

coverage 24%

api docs master

clojars [clojush "2.30.0"]

Lee Spector ([Inspector@hampshire.edu](mailto:Inspector@hampshire.edu)), started 20100227 [See version history](#). Older version history is in `old-version-history.txt`.

This is the README file accompanying Clojush, an implementation of the Push programming language and the PushGP genetic programming system in the Clojure programming language. Among other features this implementation takes advantage of Clojure's facilities for multi-core concurrency.

## Availability

[https://github.com/Inspector/Clojush/](https://github.com/Inspector/Clojush)

## Requirements

To use this code you must have a Clojure programming environment; see <http://clojure.org/>. The current version of Clojush requires Clojure 1.7.0.


Clojure is available for most OS platforms. [A good starting point for obtaining and using Clojure.](#)

## Quickstart

Using [Leiningen](#) you can run an example from the OS command line (in the Clojush directory) with a call like:

```
lein run clojush.problems.demos.simple-regression
```



 erp12 Fixed install instructions and example docs

e4dbfc5 on Mar 16

1 contributor

118 lines (85 sloc) | 3.51 KB

Raw

Blame

History



# Pysh

Push Genetic Programming in Python. For the most complete documentation, refer to the [ReadTheDocs](#).

<http://pysh2.readthedocs.io/en/latest/>

## Push Genetic Programming

Push is programming language that plays nice with evolutionary computing / genetic programming. It is a stack-based language that features 1 stack per data type, including code. Programs are represented by lists of instructions, which modify the values on the stacks. Instructions are executed in order.

More information about PushGP can be found on the [Push Redux](#) and the [Push Homepage](#).

For the most cutting edge PushGP framework, see the [Clojure](#) implementation called [Clojush](#).

## Installing Pysh

Pysh is compatible with python `2.7.x` and `3.5.x`

### Install from pip

Coming with first beta release of `pyshgp`. Check the [Roadmap](#) to get a sense of how far off this is.

# Why Push?

- Expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Supports several forms of meta-evolution
- Evolvable? At minimum, supports investigation of relations between expressiveness and evolvability

# Plush

Instruction	integer_eq	exec_dup	char_swap	integer_add	exec_if	
Close?	2	0	0	0	1	
Silence?	1	0	0	1	0	

- Linear genomes for Push programs
- Facilitates useful placement of code blocks
- Permits uniform linear genetic operators
- Allows for epigenetic hill-climbing

# Push Program Execution

- Push the program onto the exec stack.
- While exec isn't empty, pop and **do** the top:
  - If it's an instruction, execute it.
  - If it's a literal, push it onto the appropriate stack.
  - If it's a list, push its elements back onto the exec stack one at a time.

# Instructions Implemented for Most Types

- `<type>_dup`
- `<type>_empty`
- `<type>_eq`
- `<type>_flush`
- `<type>_pop`
- `<type>_rot`
- `<type>_shove`
- `<type>_stackdepth`
- `<type>_swap`
- `<type>_yank`
- `<type>_yankdup`

# Selected Integer Instructions

`integer_add integer_dec integer_div  
integer_gt integer_fromstring integer_min  
integer_mult integer_rand`

# Selected Boolean Instructions

`boolean_and boolean_xor boolean_frominteger`

# Selected String Instructions

`string_concat string_contains string_length  
string_removechar string_replacechar`

# Selected Exec Instructions

Conditionals:

`exec_if` `exec_when`

General loops:

`exec_do*while`

“For” loops:

`exec_do*range` `exec_do*times`

Looping over structures:

`exec_do*vector_integer` `exec_string_iterate`

Combinators:

`exec_k` `exec_y` `exec_s`

# Many More Types and Instructions

code\_atom code\_car print\_newline integer\_sub integer\_inc boolean\_stackdepth return\_exec\_pop vector\_integer\_eq autoconstructive\_integer\_rand boolean\_pop genome\_close\_inc string\_fromchar vector\_string\_shove zip\_yankdup genome\_new vector\_float\_yankdup exec\_yankdup vector\_integer\_shove integer\_yankdup string\_flush boolean\_swap zip\_empty exec\_shove vector\_boolean\_yank code\_eq exec\_y boolean\_yank integer\_eq genome\_silence string\_butlast code\_contains string\_conjchar code\_do\*count vector\_float\_last genome\_pop string\_substring integer\_mult code\_length vector\_integer\_dup boolean\_or code\_position boolean\_empty zip\_fromcode print\_vector\_string vector\_boolean\_swap return\_frominteger vector\_float\_pushall char\_iswhitespace code\_cdr exec\_do\*vector\_integer integer\_rand vector\_string\_replacefirst string\_first boolean\_first exec\_do\*while exec\_string\_iterate string\_indexofchar vector\_float\_replace integer\_fromstring code\_list code\_swap char\_frominteger genome\_gene\_randomize vector\_integer\_emptyvector vector\_string\_eq vector\_float\_butlast exec\_empty zip\_end? exec\_fromzipnode string\_shove vector\_boolean\_pushall zip\_insert\_left\_fromcode exec\_rot vector\_string\_concat vector\_float\_indexof code\_pop vector\_string\_subvec vector\_integer\_swap code\_subst char\_pop return\_string\_pop zip\_yank exec\_dup vector\_integer\_butlast vector\_float\_rest vector\_string\_flush boolean\_fromfloat code\_fromziprights float\_sin boolean\_flush char\_isdigit float\_lte exec\_fromziproot vector\_integer\_empty print\_code vector\_string\_stackdepth string\_reverse exec\_k vector\_integer\_yank float\_frominteger char\_rot print\_char vector\_integer\_stackdepth vector\_boolean\_concat boolean\_xor integer\_gte genome\_yankdup vector\_float\_shove vector\_integer\_take code\_quote string\_replacefirst return\_fromstring exec\_fromziplefts vector\_integer\_yankdup boolean\_shove float\_lt vector\_string\_dup vector\_string\_occurrencesof vector\_integer\_replace zip\_branch? vector\_float\_reverse float\_mod vector\_float\_subvec string\_last print\_boolean boolean\_rot vector\_string\_rest integer\_div vector\_float\_remove integer\_fromfloat integer\_lte code\_fromzipchildren environment\_end vector\_integer\_rot integer\_mod string\_concat vector\_string\_butlast genome\_swap code\_null exec\_do\*count vector\_float\_emptyvector vector\_string\_yankdup integer\_rot float\_yankdup vector\_string\_rot zip\_replace\_fromexec vector\_string\_take integer\_add vector\_integer\_occurrencesof integer\_shove genome\_dup return\_code\_pop char\_swap integer\_max return\_fromexec code\_wrap return\_float\_pop code\_flush genome\_yank zip\_shove vector\_integer\_flush vector\_integer\_subvec vector\_boolean\_indexof vector\_float\_pop vector\_string\_remove vector\_integer\_contains zip\_remove code\_append vector\_float\_eq vector\_integer\_conj string\_eq zip\_leftmost code\_yankdup code\_rot integer\_stackdepth float\_max vector\_boolean\_set zip\_append\_child\_fromexec zip\_next vector\_float\_conj zip\_fromexec string\_take zip\_left zip\_replace\_fromcode char\_stackdepth return\_fromchar genome\_eq vector\_integer\_replacefirst float\_stackdepth code\_fromziproot float\_fromchar float\_gt boolean\_dup float\_fromboolean code\_fromzipnode genome\_rot vector\_float\_replacefirst vector\_boolean\_conj vector\_boolean\_dup vector\_integer\_indexof vector\_string\_swap exec\_eq string\_emptystring string\_swap integer\_yank exec\_while float\_empty print\_vector\_boolean integer\_min exec\_swap genome\_rotate integer\_fromchar vector\_string\_yank string\_stackdepth code\_do\*range string\_replacechar char\_allfromstring vector\_integer\_rest vector\_boolean\_length char\_yank vector\_float\_empty code\_fromfloat genome\_parent2 return\_fromcode string\_pop float\_eq vector\_boolean\_empty zip\_insert\_child\_fromexec vector\_string\_last string\_nth code\_do\* return\_zip\_pop vector\_string\_pop zip\_rot vector\_integer\_nth exec\_do\*range exec\_if char\_shove zip\_down zip\_insert\_left\_fromexec code\_frominteger vector\_boolean\_remove vector\_integer\_remove boolean\_invert\_first\_then\_and genome\_flush print\_string integer\_fromboolean char\_yankdup code\_do vector\_string\_first boolean\_frominteger string\_setchar vector\_integer\_last char\_isletter genome\_gene\_dup vector\_integer\_concat print\_vector\_integer code\_map boolean\_eq float\_gte return\_fromfloat genome\_gene\_copy string\_occurrencesofchar string\_replacefirstchar print\_float boolean\_rand integer\_flush float\_shove string\_replace char\_dup float\_pop char\_eq vector\_float\_nth vector\_string\_conj integer\_gt return\_integer\_pop float\_sub vector\_integer\_length vector\_float\_set vector\_string\_indexof vector\_boolean\_rest code\_dup vector\_boolean\_shove zip\_eq float\_min boolean\_not float\_mult float\_fromstring genome\_unsilence code\_if vector\_integer\_pop vector\_boolean\_last exec\_do\*times zip\_pop zip\_rightmost float\_dec vector\_float\_contains genome\_gene\_copy\_range environment\_new exec\_do\*vector\_string code\_nthcdr string\_empty char\_empty exec\_pop vector\_integer\_set autoconstructive\_boolean\_rand vector\_float\_rot string\_yankdup exec\_do\*vector\_float string\_removechar code\_extract vector\_string\_replace vector\_float\_first genome\_parent1 return\_tagospace char\_flush vector\_float\_occurrencesof vector\_string\_emptyvector float\_add code\_stackdepth exec\_s zip\_insert\_right\_fromexec float\_dup vector\_string\_nth zip\_stackdepth vector\_integer\_reverse print\_vector\_integer char\_fromfloat code\_do\*times code\_noop zip\_swap code\_yank integer\_lt vector\_boolean\_eq genome\_stackdepth code\_fromziplefts noop\_open\_paren string\_containschar string\_yank char\_rand zip\_flush vector\_boolean\_rot float\_swap exec\_fromziprights vector\_string\_pushall vector\_string\_set vector\_boolean\_flush exec\_noop code\_size vector\_boolean\_stackdepth vector\_integer\_pushall vector\_boolean\_reverse integer\_swap string\_split vector\_boolean\_contains string\_fromboolean return\_boolean\_pop vector\_float\_dup vector\_boolean\_replace integer\_dup vector\_boolean\_nth vector\_string\_length string\_rest zip\_insert\_child\_fromcode float\_tan string\_rot string\_rand exec\_yank string\_parse\_to\_chars integer\_pop integer\_empty vector\_float\_flush vector\_float\_yank noop\_delete\_prev\_paren\_pair print\_exec zip\_append\_child\_fromcode genome\_gene\_delete code\_empty float\_inc zip\_right vector\_float\_length float\_rand integer\_dec string\_contains return\_fromboolean vector\_float\_concat vector\_float\_stackdepth exec\_do\*vector\_boolean vector\_integer\_first genome\_shove code\_rand print\_vector\_float float\_rot return\_char\_pop vector\_string\_contains vector\_boolean\_occurrencesof genome\_empty zip\_prev genome\_toggle\_silent vector\_string\_reverse zip\_dup code\_cons code\_member exec\_stackdepth float\_flush boolean\_and vector\_boolean\_butlast string\_length float\_cos string\_frominteger exec\_flush vector\_string\_empty exec\_when vector\_float\_swap genome\_close\_dec code\_insert vector\_boolean\_pop float\_div zip\_insert\_right\_fromcode code\_fromboolean vector\_boolean\_take code\_shove environment\_begin vector\_float\_take boolean\_invert\_second\_then\_and code\_container code\_nth vector\_boolean\_subvec float\_yank zip\_up vector\_boolean\_emptyvector vector\_boolean\_replacefirst string\_fromfloat vector\_boolean\_yankdup string\_dup boolean\_yankdup exec\_fromzipchildren



```
;; https://github.com/lspector/Clojush/
```

```
=> (run-push '(1 2 integer_add) (make-push-state))
```

```
:exec ((1 2 integer_add))
```

```
:integer ()
```

```
:exec (1 2 integer_add)
```

```
:integer ()
```

```
:exec (2 integer_add)
```

```
:integer (1)
```

```
:exec (integer_add)
```

```
:integer (2 1)
```

```
:exec ()
```

```
:integer (3)
```

```
=> (run-push '(2 3 integer_mult 4.1 5.2 float_add
              true false boolean_or)
      (make-push-state))
```

```
:exec ()
:integer (6)
:float (9.3)
:boolean (true)
```

In other words

- Put  $2 \times 3$  on the integer stack
- Put  $4.1 + 5.2$  on the float stack
- Put *true*  $\vee$  *false* on the boolean stack

```
=> (run-push '(2 boolean_and 4.1 true integer_div
              false 3 5.2 boolean_or integer_mult
              float_add)
      (make-push-state))
```

```
:exec ()
:integer (6)
:float (9.3)
:boolean (true)
```

Same as before, but

- Several operations (e.g., `boolean_and`) become NOOPs
- Interleaved operations

```
=> (run-push
      '(4.0 exec_dup (3.13 float_mult) 10.0 float_div)
      (make-push-state))

:exec ((4.0 exec_dup (3.13 float_mult) 10.0 float_div))
:float ()

:exec (4.0 exec_dup (3.13 float_mult) 10.0 float_div)
:float ()

:exec (exec_dup (3.13 float_mult) 10.0 float_div)
:float (4.0)

:exec((3.13 float_mult) (3.13 float_mult) 10.0 float_div)
:float (4.0)

...

:exec ()
:float (3.91876)
```

Computes $4.0 \times 3.13 \times 3.13 / 10.0$
---

```
=> (run-push '(1 8 exec_do*range integer_mult)
            (make-push-state))
```

```
:integer (40320)
```

Computes 8! in a fairly “human” way

```
=> (run-push '(code_quote
              (code_quote (integer_pop 1)
                          code_quote (code_dup integer_dup
                                          1 integer_sub code_do
                                          integer_mult)
                          integer_dup 2 integer_lt code_if)
              code_dup
              8
              code_do)
      (make-push-state))
```

```
:code ((code_quote (integer_pop 1) code_quote (code_dup
integer_dup 1 integer_sub code_do integer_mult)
integer_dup 2 integer_lt code_if))
:integer (40320)
```

A less “obvious” evolved calculation of 8!

```
=> (run-push '(0 true exec_while
              (1 integer_add true))
      (make-push-state))
```

```
:exec (1 integer_add true exec_while (1 integer_add
                                       true))
```

```
:integer (199)
```

```
:termination :abnormal
```

- An infinite loop
- Terminated by eval limit
- Result taken from appropriate stack(s) upon termination

```
=> (run-push '(in1 in1 float_mult 3.141592 float_mult)
              (push-item 2.5 :input (make-push-state)))
```

```
:float (19.63495)
```

```
:input (2.5)
```

Computes the area of a circle with the given radius:  $3.141592 \times \text{in1} \times \text{in1}$



```

(pushgp
  {:error-function
   (fn [program]
     (vec
      (for [input (range 10)]
        (let [output (->> (make-push-state)
                          (push-item input :input)
                          (run-push program)
                          (top-item :integer)))]
          (if (number? output)
              (Math/abs (float (- output
                                  (- (* input
                                       input
                                       input)
                                   (* 2 input input)
                                   input))))
              1000))))))
  :atom-generators (list (fn [] (lrand-int 10))
                          'in1
                          'integer_div
                          'integer_mult
                          'integer_add
                          'integer_sub)
  :population-size 100})

```

Set up run for target  $x^3 - 2x^2 - x$

# Auto-Simplification

- Loop:
  - Make it randomly simpler
  - Keep simpler if as good or better; otherwise revert
- GECCO-2014 poster: efficiently and reliably reduces the size of the evolved programs
- GECCO-2014 student paper: used as genetic operator
- GECCO-2017 ***GP best paper nominee***: improves generalization

SUCCESS at generation 29

Successful program: (5 4 in1 integer\_sub in1 integer\_mult  
integer\_sub integer\_div integer\_mult 6 integer\_sub integer\_add  
in1 5 integer\_sub integer\_add in1 5 integer\_add integer\_add  
integer\_mult in1 integer\_mult)

Errors: [0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0]

Total error: 0.0

History: null

Size: 24

Auto-simplifying with starting size: 24

...

step: 1000

program: (5 4 in1 integer\_sub in1 integer\_mult integer\_sub 6  
integer\_sub in1 5 integer\_sub integer\_add in1 5 integer\_add  
integer\_add in1 integer\_mult)

errors: [0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0]

total: 0.0

size: 20



DEMOS

# Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting

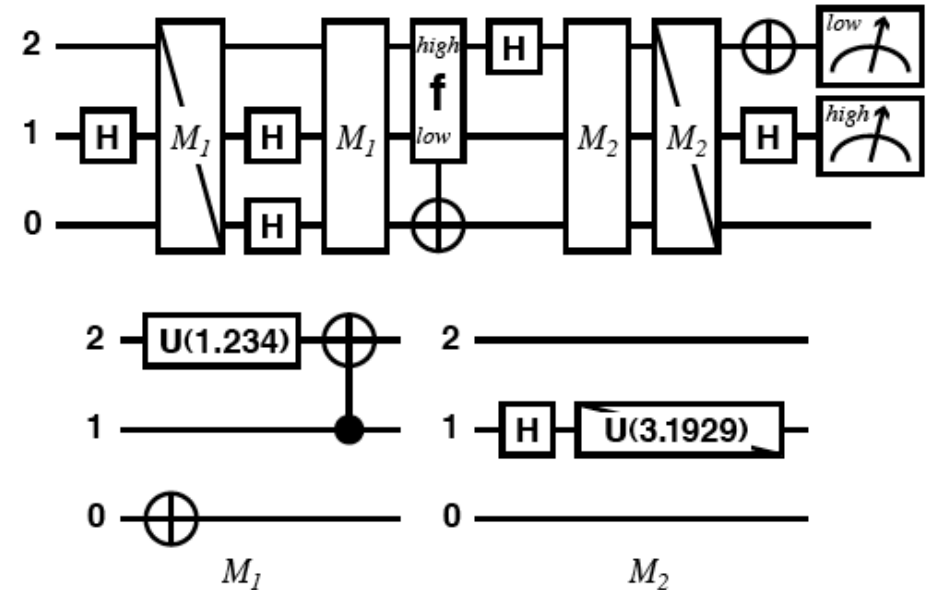
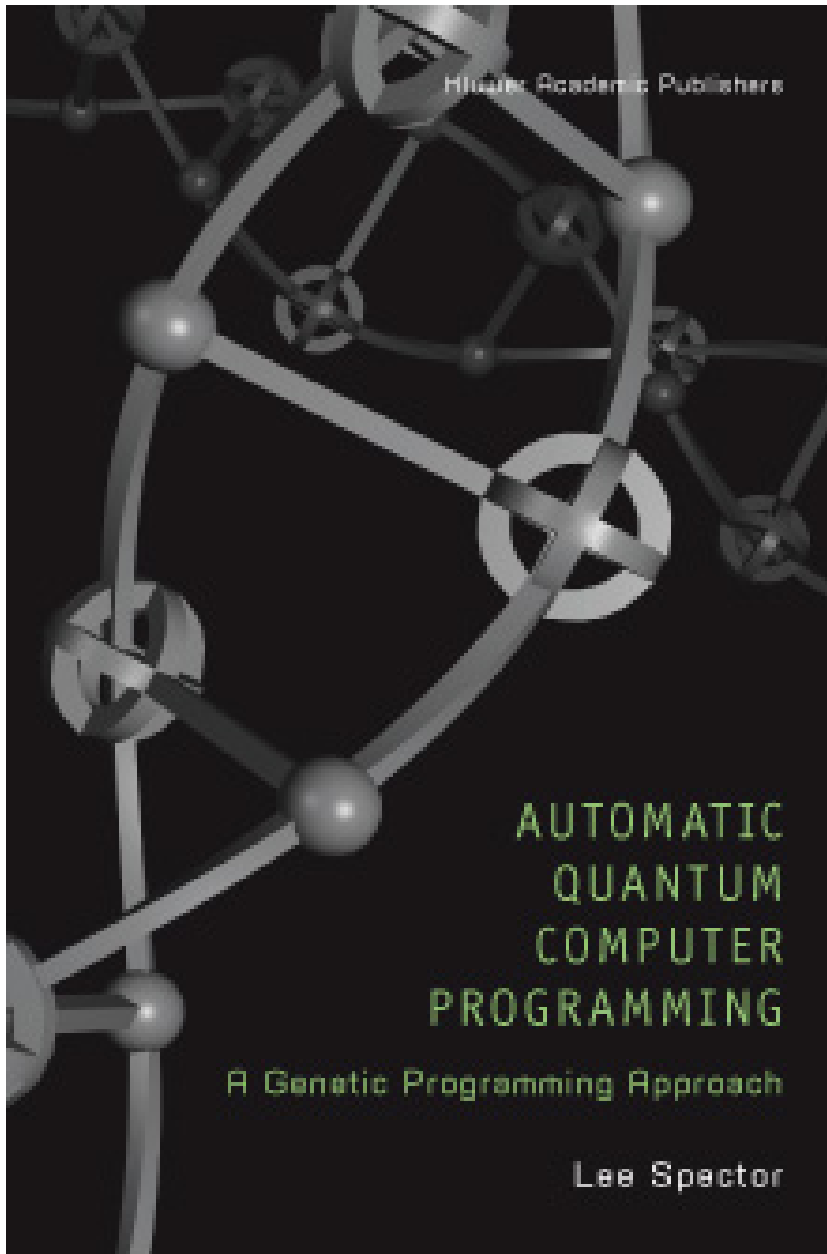


Figure 8.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with  $M_1$  and  $M_2$  standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate symbol indicates that the matrix for the gate is transposed. The "f" gate is the oracle.

Humies 2004  
GOLD MEDAL

# Genetic Programming for Finite Algebras

Lee Spector  
Cognitive Science  
Hampshire College  
Amherst, MA 01002  
lspector@hampshire.edu

David M. Clark  
Mathematics  
SUNY New Paltz  
New Paltz, NY 12561  
clarkd@newpaltz.edu

Ian Lindsay  
Hampshire College  
Amherst, MA 01002  
iml04@hampshire.edu

Bradford Barr  
Hampshire College  
Amherst, MA 01002  
bradford.barr@gmail.com

Jon Klein  
Hampshire College  
Amherst, MA 01002  
jk@artificial.com

**Humies 2008**  
**GOLD MEDAL**



# 29 Software Synthesis Benchmarks

- Number IO, Small or Large, For Loop Index, Compare String Lengths, Double Letters, [Collatz Numbers](#), Replace Space with Newline, [String Differences](#), Even Squares, [Wallis Pi](#), String Lengths Backwards, Last Index of Zero, Vector Average, Count Odds, Mirror Image, [Super Anagrams](#), Sum of Squares, Vectors Summed, X-Word Lines, [Pig Latin](#), Negative to Zero, Scrabble Score, [Word Stats](#), Checksum, Digits, Grade, Median, Smallest, Syllables
- PushGP has solved all of these except for the ones in [blue](#)
- Presented in a GECCO-2015 GP track paper

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string  
  in1 64 exec_string_iterate (integer_fromchar integer_add)  
  64 integer_mod  
  \space integer_fromchar integer_add char_frominteger  
  print_char)
```

First: Print out the header

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec string iterate (integer fromchar integer add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Second: Convert each character to an integer, sum, and add to 64.

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Third: Mod result by 64

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char frominteger
  print_char)
```

Third: Add modulus result to 32 and convert to char

# Example: Checksum

Multiple types, looping, and code blocks

Simplified solution:

```
("Check sum is " print_string
  in1 64 exec_string_iterate (integer_fromchar integer_add)
  64 integer_mod
  \space integer_fromchar integer_add char_frominteger
  print_char)
```

Fourth: Print resulting char

# Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1  
  \newline string_replacechar print_string  
  string_removechar string_length)
```



# Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1  
  \newline string_replacechar print_string  
  string_removechar string_length)
```

First: Duplicate space character and input string for use in both tasks

# Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1  
\newline string_replacechar print string  
string_removechar string_length)
```

Second: Replace spaces with newlines and print

# Example: Replace Space With Newline

Multiple types, looping, multiple tasks

Simplified solution:

```
(\space char_dup exec_dup in1  
  \newline string_replacechar print_string  
string_removechar string_length)
```

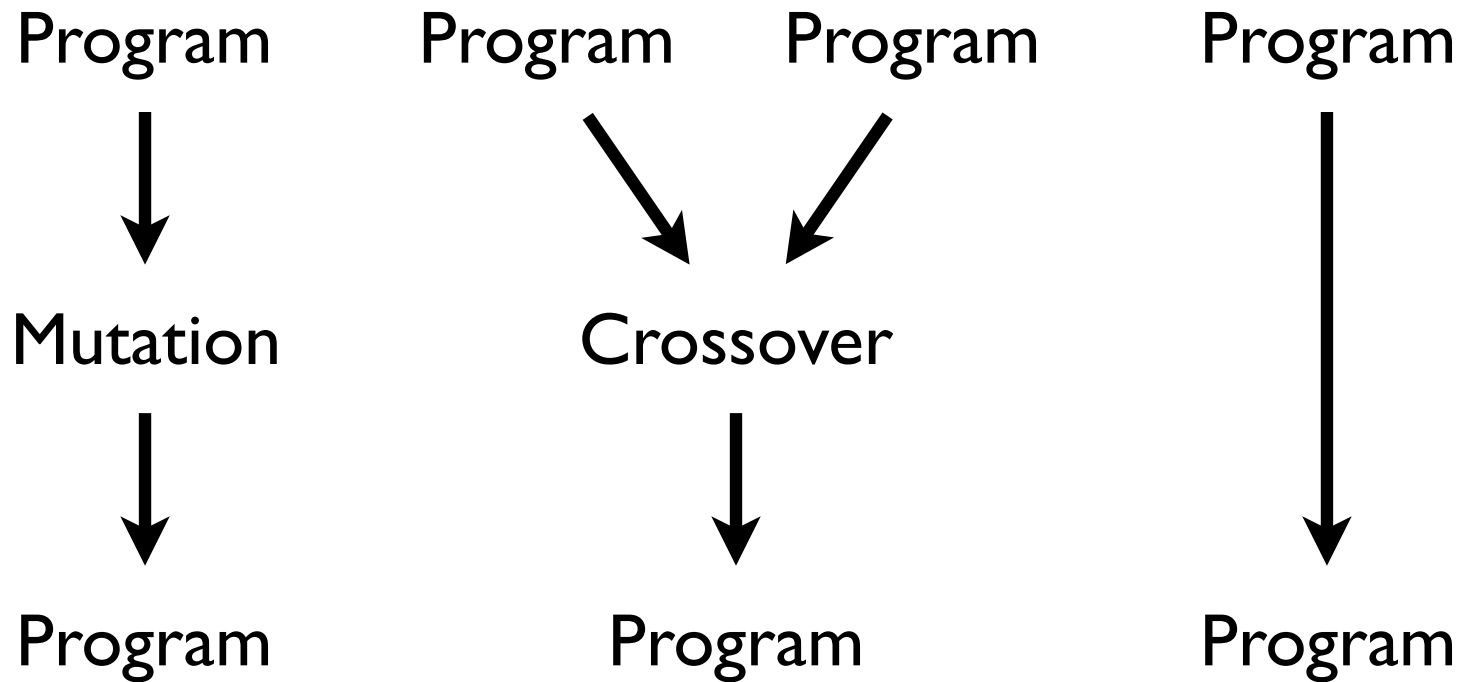
Third: Remove all spaces from second copy of input,  
and push length of result on integer stack for return

**DEMO**

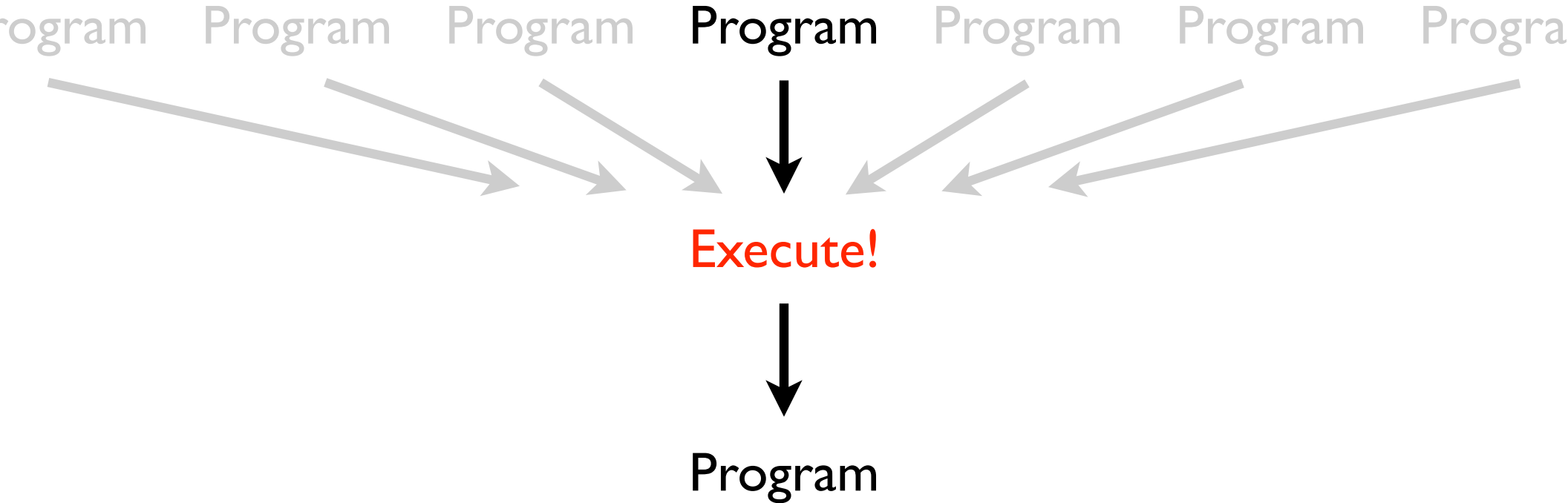
# Autoconstructive Evolution

- Individual programs make their own children
- Hence they control their own mutation and recombination methods and rates
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- In Push, experimentation with autoconstructive evolution is easy and natural

# Variation in Genetic Programming



# Autoconstruction



# Autoconstruction

- It works!
- Evolved solutions to non-trivial problems (e.g., replace-space-with-newlines)
- Only known evolved solution to string-differences
- Slower and less consistent than human engineered operators, but much more work to do
- More to explore



# Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time in random order

# Evolving Modular Programs with Push

- Via code manipulation on the code or exec stacks
- Via naming and a name stack
- Via tags, inspired by Holland's work on arbitrary identifiers with inexact matching
- Evolvability challenges abound! The relationship between evolvability, robustness, and modularity and is complex
- Push facilitates experimentation in this space

# Future Work

- Expression of variable scope and environments (implemented, but not yet studied systematically)
- Expression of concurrency and parallelism
- Applications for which expressiveness is likely to be essential, e.g. complete software applications, agents in complex, dynamic environments

# Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be unusually simple and powerful
- Push has produced significant successes and provides a rich framework for experimentation
- <http://pushlanguage.org>

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